Fault Detectability of Double Analogue Measurements using Probabilistic Analysis

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***Abstract* - Statistical multi-parameter circuit simulation is used in this work, in order to estimate the fault detection probability in cases where double analogue measurements are utilized. Theoretical analysis for the estimation of the detectability is given, based on conditional probability calculations. The proposed technique can also be applied for test measurement selection. Simulation results from the application of the method on an analogue filter circuit are given, showing a sufficient improvement over the detectability achieved by single measurements.**

***Index Terms* - analogue circuits, probabilistic analysis, analogue testing, filter design theory and applications, signal detection and estimation**

I. INTRODUCTION

Statistical circuit analysis and various "simulation-before-test" approaches, which include "fault dictionary" and other pattern matching techniques, can be used for analogue fault detection and diagnosis. The computational capabilities of today's computers are rapidly and impressively increasing and are, thus, opening up new horizons to the practical application of these methods.

Statistical methods have been proposed for analogue circuit design, optimization, fault detection and input stimulus determination [1-9]. For the statistical circuit analysis multiple circuit simulations are performed for various nominal and faulty circuit parameters under certain tolerance bounds using circuit analysis tools based on circuit simulators (for example SPICE). Monte-Carlo techniques are also utilized. The emerging "measurements" are analyzed using statistical and probabilistic mathematics, so that the results are more realistic, including possible device variations.

In this paper statistical multi-parameter circuit simulation and probability mathematics are used for the estimation of fault detectability by double analogue measurements. Following the proposed technique the fault detectability estimation can be efficiently applied for double measurements resulting in an increased value (up to 13,5%) compared to the fault detectability achieved by single measurements, while computational time is only slightly increased by less than 2%. In the following, the theoretical analysis of the proposed calculation of the probability of fault detection for double analogue measurements is described. Results from current measurements on an analogue filter circuit are then given and discussed.

II. THEORETICAL ANALYSIS

Let's denote with c the number of faulty cases on the set of faults of a circuit with known topology and parameter value deviations and with m the number of the measurements to be used. By applying the Multi Parameter Circuit Analysis shell Program MPCAP [6] for multiple Monte-Carlo analysis with the desired s samples, the arithmetic mean (μ) and the standard deviation (σ) for each faulty case and for each measurement can be obtained. The number of circuit analysis needed is large ((c+1)*s), but it can be acceptable since they are performed off-line.

The calculation of fault detectability D_k using equation (1) is based on the calculation of an overlapping integral as it has been analytically presented in our previous work [10]

$$D_k = \frac{\sum_{i=1}^{c} p_{ik}}{c} \tag{1}$$

where D_k is the measure of fault detectability for the k measurement. The higher the value of D_k, the more efficient fault detection using the k measurement can be achieved.

The estimation of fault detectability for double measurements, which is generally different from the fault detectability of each single measurement, can be calculated using a probabilistic approach. The initial idea for this approach has been briefly introduced in our previous work [6]. Let X_k be the event that the k measurement concludes to successful fault detection, in other words that the k measurement in a faulty circuit do not overlap with the k measurement in the non faulty circuit. It is obvious that the probability of this event equals to the measure of fault detectability for the k measurement calculated above (equation 1), i.e. $P(X_k) = D_k$.

The fault detectability of double measurements k and l can be expressed as the probability of the event that these two measurements together conclude to successful fault detection. This means that the distribution of either the k or the l measurement in a faulty circuit do not overlap with the k or the l measurement, respectively, of the non faulty circuit. This probability can be noted by $P\left(X_k + X_l\right)$. Since the events X_k and X_l are considered not mutually exclusive, the following equations (2) and (3) are valid [11].

$$P(X_k + X_l) = P(X_k) + P(X_l) - P(X_k \cdot X_l) \tag{2}$$

$$P(X_k | X_l) = \frac{P(X_k \cdot X_l)}{P(X_l)} \tag{3}$$

Using equation (3) in equation (2), the fault detectability $P(X_k + X_l)$ becomes:

$$P(X_k + X_l) = P(X_k) + P(X_l) - P(X_k | X_l) * P(X_l) \tag{4}$$

where $P(X_k(X_l)$ is the conditional probability of the X_k event under the assumption that the X_l event is true, and it is the only unknown coefficient.

The calculation of this conditional probability relies on the calculation of an overlapping integral similar to the one referenced above, except that the normal probability density function of the k measurement must be derived only from those Monte-Carlo cases for which the l measurement concludes to successful fault detection or, in other words, the X_l event is true. This condition may reduce the number of cases for the calculation of the normal probability density function when the $P(X_l)$ has small values, and therefore the results may become inaccurate.

It is known that the probability function has values in the range from 0 to 1 and that:

$$P(X_l) + P(\overline{X_l}) = 1 \tag{5}$$

Thus when the $P(X_l)$ has small values (<0.5), the $P(\overline{X_l})$ has a value greater than 0.5 and vice versa. Therefore, $P(X_k + X_l)$ should also be expressed as a function of the $P(\overline{X_k} | \overline{X_l})$, in order to overcome the problem of the reduced number of cases for the calculation of the normal probability density function of the conditional probability.

Equation (5) is also valid for the $(X_k + X_l)$ event as follows:

$$P(X_k + X_l) + P(\overline{X_k + X_l}) = 1 \tag{6}$$

According to DeMorgan low and equation (3) above, it is also:

$$P(\overline{X_k + X_l}) = P(\overline{X_k} \cdot \overline{X_l}) = P(\overline{X_k} | \overline{X_l}) * P(\overline{X_l}) \tag{7}$$

Combining equations (6) and (7), it can be easily proved that:

$$P(X_k + X_l) = 1 - P(\overline{X_k} | \overline{X_l}) * P(\overline{X_l}) \tag{8}$$

It must be noticed that in the above analysis for the calculation of fault detectability of double measurements the roles of k and l can be interchanged resulting in the following two alternative equations for the calculation of $P(X_k + X_l)$.

$$P(X_k + X_l) = P(X_k) + P(X_l) - P(X_l | X_k) * P(X_k) \tag{9}$$

$$P(X_k + X_l) = 1 - P(\overline{X_l} | \overline{X_k}) * P(\overline{X_k}) \tag{10}$$

In order to have more realistic and accurate results, the $P(X_k + X_l)$ must be retrieved by the calculation of the proper conditional probability using the maximum number of samples. Thus, the number of Monte-Carlo cases used for the calculation of the conditional probability is always greater or equal to s/2. The worst case would be to reduce the number of Monte-Carlo cases s to one half, in the extremely rare case where:

$$P(X_k) = P(X_l) = P(\overline{X_k}) = P(\overline{X_l}) = 0.5 \tag{11}$$

The probability $P(X_k + X_l)$ must be calculated for each faulty case and for each pair of measurements. The sum of these probabilities for all the faulty cases divided by the number of faulty cases c is a measure of the fault detectability for the double measurements k,l:

$$D_{kl} = \frac{\sum_{i=1}^{c} P_i(X_k + X_l)}{c} \tag{12}$$

The higher the value of D_{kl}, the more efficient fault detection can be obtained using the k and l measurements simultaneously. The computational time required for the calculation of the fault detection probability is negligible compared to the computational time needed for the multiple circuit analysis. The method can be extended to three or more measurements, but the probabilistic mathematics becomes much more complicated, while the results are only slightly improved, as it will be discussed in the next section.

III. RESULTS AND DISCUSSION

The above described method has been applied for the calculation of fault detectability in several circuits and has been used for test measurement selection as presented in the following. The commonly used three stage active filter circuit, shown in figure 1, triggered by a triangular 20Vp-p, 1kHz input signal, was simulated. Component parameters were considered with normal distribution with 3σ equal to 5% of their nominal values, while model parameters had the same distribution with 3σ equal to 10% of their nominal values. The set of faults consists of 32 hard faults (open-circuits and short-circuits); the measurements used were the rms values of the positive power supply current waveform and the values of the first five harmonics of its spectrum in dB, so the total number of measurements was six.

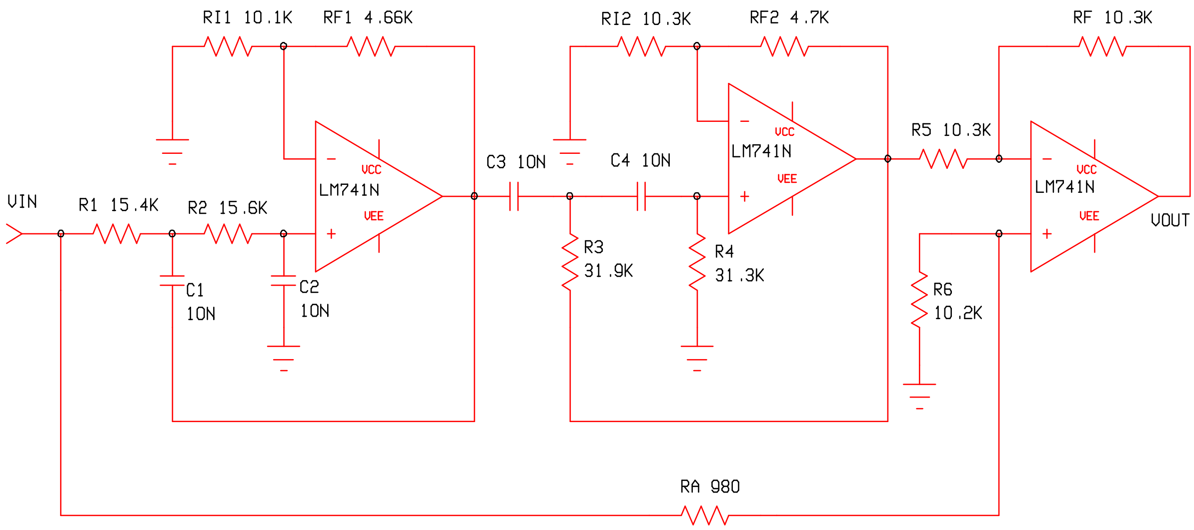

Fig. 1 The examined three stage active filter circuit of the exam ple.

Component parameters are represented by the resistor and capacitor values, while model parameters are represented, for example, by the beta values of the transistors included in the analytical opamp model. All of the 32 hard faults are affecting the values of the discrete components of the circuit. The circuit was simulated using the MPCAP shell program with 500 samples per Monte-Carlo analysis and the arithmetic mean and standard deviation matrices were obtained.

The fault detection probabilities D_k of each measurement using equation (1) are shown in table I. Measurement 1, the rms value of I_{PS}, gives the higher fault detection probability (89.4%) while measurement 2, the 1st harmonic of the I_{PS} spectrum, gives the smaller one (79.1%).

TABLE I
FAULT DETECTION PROBABI LITY OF SINGLE MEASU REMENTS OF THE THREE STAG E ACTIVE FILTER CIRC UIT EXAMPLE

K	1	2	3	4	5	6
D_k	89.4	79.1	86.7	82	83.1	83.6

Following the method described in the previous section, the probability of fault detection D_{kl} for all the possible 15 pairs of measurements were calculated as shown in table II. It must be noted that the detectability of double measurements is always greater than the detectability of each corresponding single measurement, as expected. The improvement in detectability between D_k or D_l and D_{kl} are given in table II (rows 3 and 4). This improvement ranges from 2,8% to 13,5%, showing a remarkable enhancement on the fault detectability. The double measurements (4,5) gives the highest detection probability (93.2%), while the double measurements (4,6) gives the lowest (88%). The additional computational time required for the calculation of the fault detection probability using the previous described equations to check the double measurements is only about 2% of the time required for the circuit simulations.

Using the above results for test measurements selection it is clear that if the circuit must be tested by only one measurement, then the rms I_{PS} value should be used, which gives the highest (89.4%) fault detection probability. Testing the circuit with double measurements, the third and forth harmonics of the I_{PS} spectrum (pair 4,5) must be used, increasing fault detection probability to 93.2%. This percentage difference (3.8%) seems small provided that fault detectability has already very high values (almost 90%). However its contribution is rather valuable considering that it usually helps to discriminate the most difficult to detect faulty cases.

In case where three or more measurements were used, fault detectability is only slightly improved by about 0.2%, as it was found by separately checking the number of detected faults over the 500 cases of the Monte-Carlo analysis. Thus, taking also into account the increase in the mathematical and computational complexity and that fault detectability has already very high values, the use of more than two measurements becomes inefficient.

TABLE II
FAULT DETECTION PROBA BILITY OF DOUBLE MEASUREMENTS OF THE THREE STAG E ACTIVE FILTER CIRC UIT EXAMPLE

Pair	1,2	1,3	1,4	1,5	1,6	2,3	2,4	2,5	2,6	3,4	3,5	3,6	4,5	4,6	5,6
D_{kl}	92.6	92.4	92.9	92.3	92.2	92.6	88.1	89.5	88.2	92.2	92.7	92.2	93.2	88	92.9
D_{kl} - D_k	3.2	3	3.5	2.9	2.8	13.5	9	10.4	9.1	5.5	6	5.5	11.2	6	9.8
D_{kl} - D_l	13.5	5.7	10.9	9.2	8.6	5.9	6.1	6.4	4.6	10.2	9.6	8.6	10.1	4.4	9.3

IV. CONCLUSIONS

Statistical multi-parameter circuit analysis and probabilistic mathematics has been efficiently utilized for the estimation of fault detectability of double analogue measurements in a simulation-before-test approach. The additional computational time is negligible compared to the computational time needed for the multiple circuit analysis, while fault detectability is enhanced up to 13,5% compared to the one calculated by single measurements. In cases where fault detectability has already very high values, the benefit is not focused on the increase of its value but on the fact that the additional faults that are distinguished lay among the most difficult to detect faulty cases.

The results were efficiently used to select the pair of measurements with the highest detectability among a given set of measurements. The work could also be used for input stimulus selection, by applying the described procedure for various inputs. The method could be extended to three or more measurements but the fault detectability enhancement seems to be insignificant compared to the additional mathematical and computational complexity.

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