An Automatic System for Aircraft Collision Avoidance in Free Flight: The 3-D Case.

Manolis A. Christodoulou
Senior Member IEEE
Department of Electronic and Computer Engineering, Technical University of Crete
Chania 73100, Greece
manolis@systems.tuc.gr

Sifis K. Kodaxakis
Student Member IEEE
Department of Electronic and Computer Engineering, Technical University of Crete
Chania 73100, Greece
sifis@systems.tuc.gr

Yiannis S. Boutalis
Member IEEE
Department of Electrical and Computer Engineering, Democritus University of Thrace,
Xanthi 67100, Greece
ybout@ee.duth.gr

***Abstract*— In this paper, we consider optimal resolution of air traffic (AT) conflicts. Aircrafts are assumed to cruise within a free altitude layer and are modeled in 3-D dimensions with variable velocity and proximity bounds. Aircrafts cannot get closer to each other than a predefined safety distance. We consider the problem of solving conflicts arising among several aircrafts that are assumed to move in a shared airspace.**

For such systems of multiple aircrafts, we are interested in optimal path, i.e. we want to minimize the total flight time by avoiding all possible conflicts. This paper proposes one formulation of the multi-aircraft conflict avoidance problem as a mixed integer non linear programming problem. In our case, only velocity changes are admissible as maneuvers. Nevertheless in subsequent work we will be checking for simultaneous velocity and heading angle changes too. Simulation results for realistic aircraft conflict scenarios are provided.

Index Terms – Free Flight, Collision Avoidance, Non Linear Mixed Integer Programming.

I. Introduction

A. *The concept of Free Flight*

SUPPOSE that sometime this year two US airliners find themselves on a collision course. An air traffic controller relying on rapidly deteriorating 1960s-vintage equipment must distinguish the two planes from others on his radar screen and make a mental calculation about the likelihood of a crash. Though his radar display is two-dimensional, the controller must envision the planes [1] flight paths in three dimensions, then project the paths through time. He must take into account that, say, one plane is turning sharply and rapidly descending, while the other is slowly climbing. Alarmed, he'll quickly get on his radio to instruct one of the pilots to change course. Only if his message isn't garbled or drowned out or misunderstood will an accident be averted.

Now imagine the same scenario 15 years from now. Both planes carry satellite-based navigational equipment that identifies their positions with precision far outstripping radar's. Instead of passing information by voice, pilots rely on digital communications gear that automatically transmits a constant flow of data about the planes [1] location, direction, and speed to controllers and other nearby aircrafts. Surveillance and data processing equipment on the ground and aboard the aircraft projects the planes' flight paths over time, instantaneously making the same calculations that the 2005 controller struggled to do in his head. Long before the two aircrafts seriously threaten each other, cockpit displays in both planes warn the pilots of the potential conflict and recommend course changes.

Welcome to "free flight," the aviation community's term for changes that constitute the most significant development in air traffic management since the invention of radar 70 years ago. Under free flight, many tasks now carried out by air traffic controllers will be automated, and some of the authority that controllers possess will be shifted to pilots. Not surprisingly, the engine driving free flight is digital technology, which has laid bare the obsolescence of the current air traffic control system.

B. *Free Flight Advocates*

The Federal Aviation Administration, which oversees the US air traffic control system, has embraced the concept but may have a hard time persuading **Congress to allocate the many billions of dollars that free flight will probably cost.** Thanks to past debacles in developing air traffic management technology, the FAA's credibility is notoriously low, to the point that proposals for privatizing the agency are filtering through Congress.

C. *Reasons for Free Flight*

Airlines aren't the only potential beneficiaries. Paul Fiduccia, president of the Small Aircraft Manufacturers Association, thinks that **free flight soon could help make flying a small plane almost as simple as driving a car.** The result, Fiduccia believes, is that general aviation for personal and business use could enjoy an upsurge in popularity after 15 years of stagnation. "What we're trying to do is feed the fruits of the digital revolution into airplanes," Fiduccia says. "If we do, you'll be able to fly a plane after a couple months of training. You'll have the same operational simplicity that you have in a car with cruise control. You get the plane in the air, you sit there, and you talk with your passengers for a few hours until you get to where you're going."

The FAA sees free flight's projected efficiencies as a way of coping with growth in passenger traffic, which the agency projects - perhaps overoptimistically - at 7 percent a year over the next decade. Free flight might reduce congestion at the nation's most crowded airports by enabling planes to land with less spacing between them. The National Air Transportation Association's Coyne thinks free flight could even eliminate the need for air traffic controllers, **saving the federal government up to $5 billion a year**. While other officials are far less sanguine about the safety of a system shorn of all controllers, some believe free flight would make likely a downsizing from the current workforce of 17,000 controllers. Nevertheless, even that prospect has not led the controllers union, the National Air Traffic Controllers Association, to oppose free flight. Instead, union leaders have expressed tempered enthusiasm.

D. Phases of Free Flight

The task force's report, issued last October, calls for the introduction of free flight **in three phases spread over a minimum of five years.** In the first phase, extending through 2005, air traffic controllers would yield some of their authority and enter into a more collaborative relationship with airlines and pilots. Rule changes that do not require extensive investments in new technology would be instituted. For example, when a busy airport such as Chicago's O'Hare experiences a storm, controllers typically decide the order in which all of an airline's planes land. Chicago may be the final destination of most passengers in one plane, while half the passengers in another plane may be hoping to make connecting flights. Controllers, however, aren't aware of such passenger configurations, and, in any event, they're trained to sequence landings on a first-come, first-served basis. They may allow the plane filled with passengers bound for Chicago to land while the other plane circles overhead. **Under free flight, controllers would simply inform each airline how many landing slots it could use during the storm, and the airline would decide the order in which its planes land. That change alone would save passengers thousands of wasted hours and airlines many millions of dollars**.

Under the second phase, projected from 2005 to 2010, **airliners no longer would be limited to the rigid flight paths now prescribed by the FAA. Instead, they could choose the routes that best fit their needs.** A cargo plane might choose a route that maximizes speed or fuel efficiency, even if that means flying through a storm. A passenger plane flying between the same two airports might choose to ensure passenger comfort by avoiding the storm at a sacrifice in speed. While controllers would continue to have responsibility for keeping aircrafts from colliding, they would be instructed to approve all proposed routes as long as the planes maintain "separation" - the aviation term for collision avoidance. To do this, controllers would need digital "conflict probe" technology that tracks planes' projected courses and warns of potential conflicts.

It's in the third phase, optimistically projected to begin in 2010 and end perhaps a decade later, **that free flight's full potential could be realized.** Pilots no longer would need to ask controllers for permission to make route changes, and they'd even take on some responsibility for separation. Each plane would be considered to be surrounded by two hockey-puck shaped volumes of space, one inside the other. When conflict probe technology detects an intruder entering a plane's larger volume, known as the "alert zone," the two pilots and the ground controller would be warned to consider evasive action, while the smaller volume, called the "protected zone," would be considered inviolate. The size of a plane's zones would depend on the accuracy of its technology and its performance capacity: the more advanced the plane, the smaller the zones.

Though it's too early to predict the size of the zones, they almost certainly would be much smaller than the five miles of horizontal separation that the FAA now mandates for planes in mid-flight. Planes would be free to fly at speeds and altitudes that maximize their performance, instead of being hung up behind slower planes on prescribed air routes. Today controllers often begin shunting aircrafts into line for landing at busy airports when the planes are as far as 600 miles from their destination, but digital sequencing tools would enable planes to fly at higher, more fuel-efficient altitudes until as close as 90 miles before landing.

II. Proposed Free Flight Scenario

A. Conflict Resolution Strategy

Many approaches have been proposed in the last few years to address the conflict resolution problem when many aircrafts are involved; **a complete overview of these approaches with a complete bibliography may be found in [2].** For an extensive study on the impact of Free Flight on safety we refer the reader to the work developed at NASA Ames by Bilimoria [3], in which is proved that the Free Flight environment is safer for the current traffic in terms of possible conflict respect to the current airspace structure.

The approach proposed in this paper involves centralized, numerical optimization. We consider the problem of resolving conflicts arising among many aircrafts following a cooperative approach i.e. all aircrafts involved in a conflict are able to communicate intent with each other, so that they may follow an agreed upon maneuver (such as change velocity) which is proven a-priori to be safe. The communication between aircrafts does not currently exist, although in emergency situations aircrafts can currently communicate through an emergency radio frequency, but it will in the near future with the proposed Automatic Dependence Surveillance – Broadcast (ADS-B), in which each aircraft broadcasts to all other aircrafts in its vicinity its current state as well as intent in the form of its two proposed way points.

The algorithms of this paper do not require any additional structure to the airspace than what currently exists. Each aircraft is surrounded by two virtual hockey pucks, the protected zone and the alert zone, shown in Figure 1. **A conflict or loss of separation between two aircrafts occurs whenever the protected zones of the aircrafts overlap.** The radius and the height of the en-route protected zone are currently about 2,5 nautical miles and 2,000 ft respectively. However, it has been proposed in [4] that for true 3-dimensional free flight, protected zones need to be spheres of radius about 3-5 miles. The size of the alert zone depends on various factors including airspeed, altitude, accuracy of sensing equipment, traffic situation, aircraft performance and average human and system response times. The alert zone should be large enough to allow a comfortable system response but also small enough in order to avoid unnecessary conflicts. What is more, the size of the protected zone is direct reflection of the position determination accuracy.

For the algorithms in this paper, we have considered aircrafts randomly distributed on a three-dimensional spherical alert zone of radius 67,1 nautical miles and a spherical protected zone of radius d/2 = 2,8 nautical miles.

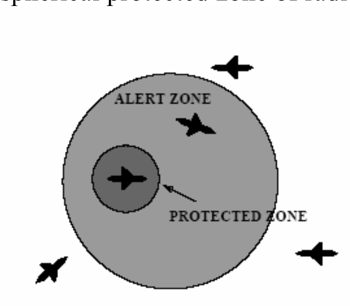

Fig. 1. Aircraft zones

The approach presented in this paper, is based on the following central assumptions:

- **Aircrafts are assumed to cruise in a non fixed altitude layer.** The task of each vehicle is to reach a given goal configuration from a given start configuration.
- **All interacting aircrafts cooperate towards optimization of a common goal, as agents in the same team.** The common goal is to reach the final configuration while avoiding all possible conflicts. This applies to all aircrafts in the same airspace, defined as a zone in which they can exchange information on positions, velocities and goals.
- We study aircraft maneuvers consisting of **instantaneous velocity changes.**

The problem of finding the shortest conflict-free paths can be modeled as a **Mixed Integer Programming (MIP) problem, which may be solved using optimization tools such as CPLEX [5]**.

The simplicity of the model allows us to manage a large number of aircrafts in the same airspace. Furthermore, due to the efficient computations used to solve the problem, we can rerun the problem at regular sample times to generate a feedback control law. Conflict avoidance constraints that are considered in this problem, are based on geometric constrains.

B. Problem Statement

We consider a finite number n of aircrafts sharing the same airspace; each aircraft is an autonomous vehicle that flies in a ***3-dimensional plane***. Each aircraft has an initial and a final desired configuration and the same goal which is to reach the final configuration in minimum time while avoiding conflicts with other aircrafts. A conflict between two aircrafts occurs if they are closer than a given distance d.

As we mentioned before, by considering the aircraft as a moving sphere of radius d/2, the condition of non conflict between aircrafts is equivalent to the condition of non intersection of the spheres. To gain a quick understanding of this problem, let's take a look at what happens when two spheres are touching. As we can see in the illustration at the Figure 3.2, the radius of each sphere now also defines the distance its center to the opposite sphere's skin. So, given this condition, the distance between the centers would be equal to Radius1+Radius2. If the distance were greater, the two spheres would not touch. If it where less, the spheres would intersect. In the following we refer to those spheres as the *safety sphere* of the aircraft.

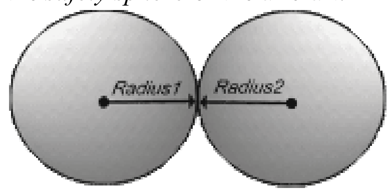

Figure 2. Spheres in touch

Aircrafts are identified in a 3-D coordinate system representing the center of the sphere (position) and 2 angles representing the space direction, by a 5-dimensional vector as follows:

$$(x, y, z, \theta, \phi) \in IRxIRxIRxS^1xS^1 .$$

Let $(x_i(t), y_i(t), z_i(t), \theta_i(t), \phi_i(t))$ be the configuration of the i-th aircraft at time t; A conflict between aircrafts i and j occurs if for some value t,

$$\sqrt{(x_i(t)-x_j(t))^2+(y_i(t)-y_j(t))^2+(z_i(t)-z_j(t))^2} < d. \qquad (1)$$

since: (radius1+radius2) = (d/2+d/2) = d.

To avoid possible conflicts, we allow aircrafts to change the velocity of flight but the direction of motion remains fixed. We will refer to this case as the Velocity Change problem (VC problem).

Each aircraft is allowed to make a maneuver, at time t = 0, to avoid all possible conflicts with other aircrafts. We assume that no conflict occurs at time t = 0;

Let's define by q_i the velocity change of the i-th aircraft.

The problem consists in **finding a minimum velocity change q_i, for each aircraft, to avoid any possible conflict while deviating as little as possible from the original flight plan**. The problem considered can be formulated as a mixed integer nonlinear optimization problem with nonlinear constraints and some Boolean variables. In the following section we formulate conflict avoidance constraints that are nonlinear in those velocity variations q_i.

C. Conflict Avoidance Constraints for the VC problem

In this section we obtain, by geometrical considerations, the conflict avoidance constraints for the VC problem. The VC problem consists of aircrafts that fly along a given fixed direction and can maneuver only once with a velocity variation. The i-th aircraft changes its velocity by a quantity q_i that can be positive (acceleration), negative (deceleration) or null (no velocity variation). Each aircraft has upper and lower bounds on the velocity $\upsilon_i : \upsilon_{i,\min} \le \upsilon_i \le \upsilon_{i,\max}$. For commercial flights, during en route flight we usually have $\frac{\upsilon_{i,\max} - \upsilon_{i,\min}}{\upsilon_{i,\min}} \le 0.1$. The problem then is to find an admissible value of q_i, for each aircraft, such that all conflicts are avoided and such that new velocity satisfies the upper and lower bounds. (For the VC problem in 2 dimensions, i.e. aircrafts flying on a plane, see [6]). Hence, given the initial velocity υ_i, after a velocity variation of amount q_i the following inequalities must be satisfied:

$$\upsilon_{i,\min} \le \upsilon_i + q_i \le \upsilon_{i,\max} \tag{2}$$

We will originally restrict to the case of two aircrafts, to obtain conflict avoidance conditions and then we will consider the general case of *n* aircrafts. Consider two aircrafts denoted by 1 and 2, respectively and let $(x_i, y_i, z_i, \theta_i, \phi_i)$, i = 1, 2 be the aircrafts positions and directions of motion and υ_i be the initial velocities.

Referring to Figure 3, we consider the two velocity vectors:

$$\vec{\upsilon}_1 = \begin{bmatrix} (\upsilon_1 + q_1)\sin\phi_1 \cos\theta_1 \\ (\upsilon_1 + q_1)\sin\phi_1 \sin\theta_1 \\ (\upsilon_1 + q_1)\cos\phi_1 \end{bmatrix}; \tag{3}$$

$$\vec{\upsilon}_2 = \begin{bmatrix} (\upsilon_2 + q_2)\sin\phi_2 \cos\theta_2 \\ (\upsilon_2 + q_2)\sin\phi_2 \sin\theta_2 \\ (\upsilon_2 + q_2)\cos\phi_2 \end{bmatrix}; \tag{4}$$

and the difference vector:

$$\vec{\upsilon}_1 - \vec{\upsilon}_2 = \begin{bmatrix} (\upsilon_1 + q_1)\sin\phi_1 \cos\theta_1 - (\upsilon_2 + q_2)\sin\phi_2 \cos\theta_2 \\ (\upsilon_1 + q_1)\sin\phi_1 \sin\theta_1 - (\upsilon_2 + q_2)\sin\phi_2 \sin\theta_2 \\ (\upsilon_1 + q_1)\cos\phi_1 - (\upsilon_2 + q_2)\cos\phi_2 \end{bmatrix}; \tag{5}$$

where, $0 \le \theta_i \le 2\pi$ and $0 \le \phi_i \le \pi$.

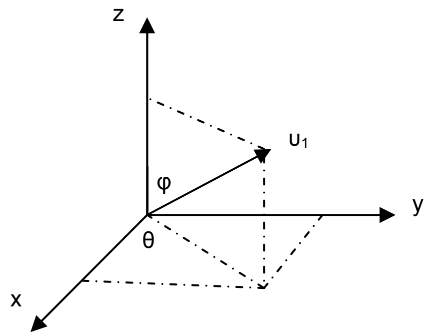

Figure 3.: Velocity vector

The non-parallel straight lines that are tangent to the spheres of both aircrafts, localize a segment in the direction on motion of 1 (refer to Figure 4). We refer to this segment as the ***cone*** of aircraft 2 along the direction of 1. A conflict occurs if the aircraft 1 and its safety sphere intersect the cone generated by aircraft 2, or vice versa.

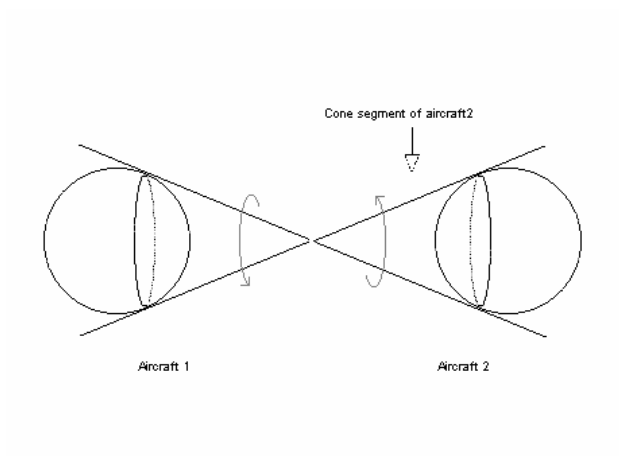

Figure 4. Cone sections between two moving spheres

We refer to the above case, by examining the motion of two spheres in 2-dimensions because of symmetry, in the plane defined by vector $\vec{s}$, which represents the distance of the two spheres centers, and by $\vec{\upsilon}_{12}$ the vector of relative speed of motion among the 2 flying aircrafts. (Refer to Figure 5).

Consider now the two non-parallel straight lines that are tangent to the discs of both aircrafts. Let α be the angle between the first straight line and the horizontal axis, and ω be the angle between the vector $\vec{\upsilon}_{12}$ of relative speed and the vector $\vec{s}$ which represents the distance of the two spherical centers.

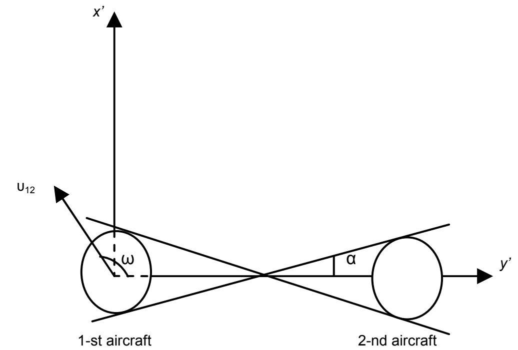

Figure 5. The two non parallel straight lines tangent to the safety discs of radius d/2 for two aircraft at distance A_{12}

If ω is the angle between vectors $\vec{\upsilon}_{12}$ and $\vec{s}$ we have $\cos\omega = \frac{\upsilon_{12} \bullet s}{|\upsilon_{12}||s|}$, $\vec{s} = \begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{bmatrix}$.Since $\tan\omega = \frac{\pm\sqrt{1-\cos^2\omega}}{\cos\omega}$ in the case of positive sign no conflict occurs if:

$$\frac{\sqrt{1-\left(\frac{\upsilon_{12} \bullet s}{|\upsilon_{12}||s|}\right)^2}}{\frac{\upsilon_{12} \bullet s}{|\upsilon_{12}||s|}} \geq \tan(\alpha) \quad \text{or} \quad \frac{\sqrt{1-\left(\frac{\upsilon_{12} \bullet s}{|\upsilon_{12}||s|}\right)^2}}{\frac{\upsilon_{12} \bullet s}{|\upsilon_{12}||s|}} \leq \tan(-\alpha) \qquad (6,7)$$

To obtain non conflict constraints for n aircrafts we need to consider the non conflict conditions described by (6) and (7) for all possible pairs of aircrafts. Let's consider the pair of aircrafts (i, j). We have to distinguish between two possible cases: 1) $\upsilon_{ij} \bullet s < 0$ and 2) $\upsilon_{ij} \bullet s > 0$. We also have $\tan(\alpha) = \frac{d/2}{A_{ij}/2} = \frac{d}{A_{ij}}$, where A_{ij} is the distance between the two aircrafts i and j. So, we obtain the following groups of constraints:

Case 1: $\upsilon_{ij} \bullet s < 0$ <u>AND</u> $\tan\omega$ has "positive sign"

$$\left\{ \begin{array}{l} \upsilon_{ij} \bullet s \leq 0 \\ |\upsilon_{ij}||s| * \sqrt{1-\left(\frac{\upsilon_{ij} \bullet s}{|\upsilon_{ij}||s|}\right)^2} - \frac{d}{A_{ij}} * (\upsilon_{ij} \bullet s) \leq 0 \end{array} \right\} \qquad (8)$$

or

$$\left\{ \begin{array}{l} \upsilon_{ij} \bullet s \leq 0 \\ -|\upsilon_{ij}||s| * \sqrt{1-\left(\frac{\upsilon_{ij} \bullet s}{|\upsilon_{ij}||s|}\right)^2} - \frac{d}{A_{ij}} * (\upsilon_{ij} \bullet s) \leq 0 \end{array} \right\} \qquad (9)$$

Case 2: $-\upsilon_{ij} \bullet s > 0$ <u>AND</u> $\tan\omega$ has "positive sign"

$$\left\{ \begin{array}{l} -\upsilon_{ij} \bullet s \geq 0 \\ -|\upsilon_{ij}||s| * \sqrt{1-\left(\frac{\upsilon_{ij} \bullet s}{|\upsilon_{ij}||s|}\right)^2} + \frac{d}{A_{ij}} * (\upsilon_{ij} \bullet s) \leq 0 \end{array} \right\} \qquad (10)$$

or

$$\left\{ \begin{array}{l} -\upsilon_{ij} \bullet s \geq 0 \\ |\upsilon_{ij}||s| * \sqrt{1-\left(\frac{\upsilon_{ij} \bullet s}{|\upsilon_{ij}||s|}\right)^2} + \frac{d}{A_{ij}} * (\upsilon_{ij} \bullet s) \leq 0 \end{array} \right\} \qquad (11)$$

These two groups of constraints will be included in the model as or-constraints. **All constraints obtained are nonlinear in the variable that represents the velocity change q_i.** To conclude with the problem formulation we must consider the upper and lower bounds in (2) that are already linear in q_i.

As noted earlier, only one set of constraints will be used in our model for each instance. Thus, subject to which of cases (1 or 2) holds true, we use the first set (8-10) or the second set (9-11) of equations.

In the case of negative sign, for the tangent $\tan\omega$, by using analogous reasoning, no conflict between all n aircrafts occurs if:

Case 1: $\upsilon_{ij} \bullet s < 0$ <u>AND</u> $\tan\omega$ has "negative sign"

$$\left\{ \begin{array}{l} \upsilon_{ij} \bullet s \leq 0 \\ -|\upsilon_{ij}||s| * \sqrt{1-\left(\frac{\upsilon_{ij} \bullet s}{|\upsilon_{ij}||s|}\right)^2} - \frac{d}{A_{ij}} * (\upsilon_{ij} \bullet s) \leq 0 \end{array} \right\} \qquad (12)$$

or

$$\left\{ \begin{array}{l} \upsilon_{ij} \bullet s \leq 0 \\ |\upsilon_{ij}||s| * \sqrt{1-\left(\frac{\upsilon_{ij} \bullet s}{|\upsilon_{ij}||s|}\right)^2} - \frac{d}{A_{ij}} * (\upsilon_{ij} \bullet s) \leq 0 \end{array} \right\} \qquad (13)$$

Case 2: $-\upsilon_{ij} \bullet s > 0$ <u>AND</u> $\tan\omega$ has "negative sign"

$$\left\{ \begin{array}{l} -\upsilon_{ij} \bullet s \geq 0 \\ |\upsilon_{ij}||s| * \sqrt{1-\left(\frac{\upsilon_{ij} \bullet s}{|\upsilon_{ij}||s|}\right)^2} + \frac{d}{A_{ij}} * (\upsilon_{ij} \bullet s) \leq 0 \end{array} \right\} \qquad (14)$$

$$\left\{ \begin{array}{l} -\upsilon_{ij} \bullet s \geq 0 \\ -|\upsilon_{ij}||s| * \sqrt{1-\left(\frac{\upsilon_{ij} \bullet s}{|\upsilon_{ij}||s|}\right)^2} + \frac{d}{A_{ij}} * (\upsilon_{ij} \bullet s) \leq 0 \end{array} \right\} \qquad (15)$$

Only one of these sets of constraints will be used in our model for each instance. Thus, according to which of two cases (1 or 2) holds true, we will use the first set (12 and 14) or the second set (13 and 15) of equations.

If the goal of each aircraft is to avoid all possible conflicts in minimum time then we want to maximize the value of q_i such that if q_i is negative we minimize the admissible deceleration.

In order to formulate this as a minimization problem we choose:

$$\sum_{i=1}^{n} - q_i \tag{16}$$

as the cost function.

Obviously a solution to the conflict problem does not always exist, for example in the case of head-to-head conflict a change of velocity is not sufficient to solve the problem.

D. Problem Formulation

The set of constraints obtained in the above sections are nonlinear in the decision variables q_i for the VC problem. Because we have a very large number of constraints that increases exponentially with the number of aircrafts involved, it is imperative that we use a software optimization package in order to solve it. There are, indeed, many such tools available. We have chosen the GAMS software package (www.gams.de), which is essentially a front end for solvers such as CPLEX, dicopt etc. Its friendly interface and accessibility make it an ideal tool for the user who does not wish the full processing power of professional high-end products.

We now show how the above set of constraints should be recast as mixed integer nonlinear constraints suitable for standard optimization software such as CPLEX. We assume that the reader is familiar with the basics of linear and non-linear optimization problems.

As any other optimization package, GAMS, requires that the constraints present for any problem are all satisfied simultaneously (together with the constraints). In other words GAMS is able to solve optimization problems of the form:

$$\min f(x) \tag{17}$$

such that

$$g(x) \le 0 \tag{18}$$

where $f(x)$ is a function of n real variables $x = (x_1, x_2, ..., x_n) \in R^n$ and is subject to a set of inequality constraints $g(x) \le 0$ ($g_j(x) \le 0$, j = 1, 2, p). This means that the constraints $g_j(x)$ must be all valid at the same time (g1 AND g_2 AND … AND g_p). Clearly in our case, where we have or-constraints, a reformulation is necessary. We therefore shall have to introduce new Boolean variables to convert these or-constraints to and-constraints. A simple example will be presented for comprehensive purposes.

Let us assume that we have the following sets of constraints similar to the conflict avoidance constraints described in the previous sections:

$$\begin{cases} c_1 \le 0 \\ and \\ c_2 \le 0 \end{cases} \tag{19}$$

OR

$$\begin{cases} c_3 \le 0 \\ and \\ c_4 \le 0 \\ and \\ c_5 \le 0 \end{cases} \tag{20}$$

OR

$$\begin{cases} c_6 \le 0 \\ and \\ c_7 \le 0 \end{cases} \tag{21}$$

where the terms c_i, $i = 1,...,7$ are linear or nonlinear expressions in the decisions variables.

The way to transform these or-constraints is to introduce Boolean variables [7]. Let f_k with k=1, 2, 3 be a binary number that becomes zero when one of the or-constraint is active and 1 otherwise (i.e. $f_1 = 0$ if constraints c_1 and c_2 are active, $f_1 = 1$ otherwise). Letting M be a large arbitrary number, the previous set of constraints is equivalent to:

$$\begin{aligned} & c_1 - Mf_1 \le 0 \\ & c_2 - Mf_1 \le 0 \\ & c_3 - Mf_2 \le 0 \\ & c_4 - Mf_2 \le 0 \\ & c_5 - Mf_2 \le 0 \\ & c_6 - Mf_3 \le 0 \\ & c_7 - Mf_3 \le 0 \\ & f_1 + f_2 + f_3 \le 2 \end{aligned} \tag{22}$$

The above constraints are all and-constraints so we have overcome the previous difficulty. It is however obvious that now we are faced with a so-called Mixed Integer Programming (MIP) problem [8], because we have two different kinds of variables: normal variables that can take any value and binary variables ($f_1 \ldots f_3$) that can only take the values 0 or 1. MIP problems are considerably more complex than both the Pure Integer Programming problems (where the decision variables can only take binary values) and the classic LP or NLP problems.

III. Simulation And Case Studies

A. Introduction

As shown, in the previous sections, given the initial and the final positions and the goal configurations of aircrafts we can easily obtain a mixed integer nonlinear problem, for solving the 3-D VC conflict avoidance problem. In this section we report the results obtained using CPLEX to solve the VC problem. We considered aircrafts randomly distributed on a sphere of radius 67.1 nautical miles. We consider a non-symmetric case.

The initial configuration of every aircraft consists of its velocity and its 2 heading angles at the point of entry in the sphere, while its final configuration consists of its velocity and its 2 heading angles at the point of exit in the sphere. Such kind of points, have been chosen randomly for a more realistic scenario.

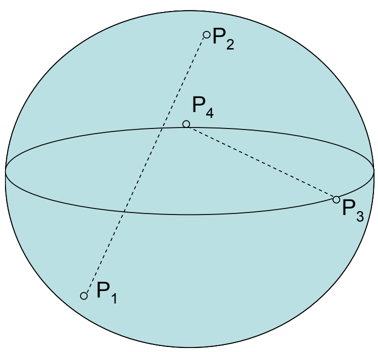

Figure 6. Initial and final configuration points for 2 aircrafts into the control sphere

Basically, **collision detection** is a system program that determines whether two objects will collide inside the sphere. If there are more than two objects, then we consider all possible pairs of objects.

If a collision occurs, the motion of objects should be reformulated. Perhaps they should move just enough to touch each other (sphere wise). Our calculations take the following order:

- Future position computation
- Possible collision detection
- Collision handling

In collision detection we usually want to know whether two objects intersect. Consider now, the prediction problem as a dynamic one. Two objects are moving relative to one another. Their positions are functions of time. We want to know exactly when they collide, if it so happens. What we want is to define some sort of relationship between the two objects that changes as a function of time. Here's what this kind of prediction problem would involve: Referring to Fig. 6, the initial configuration points for aircrafts 1,2 are $P_1(x_1,y_1,z_1)$ and $P_3(x_3,y_3,z_3)$ respectively, while the final configuration points are $P_2(x_2,y_2,z_2)$ and $P_4(x_4,y_4,z_4)$ respectively. Hence, the parametric equations for the line "l" through P_1 and P_2 are the following:

$$\begin{cases} x = x_1 + a_1\lambda \\ y = y_1 + a_2\lambda \\ z = z_1 + a_3\lambda \end{cases} \qquad (23)$$

where, $a = (a_1, a_2, a_3) = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$ and "λ" is a variable.

Since aircraft 1 is moving in a straight line with a standard velocity υ_1, we have a linear equation. In time "t" aircraft 1 "travels" an interval "s", so we have:

$$s = \upsilon_1 t \qquad (24)$$

Using the fact that in the same time "t", we have:

$$s = \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} \qquad (25)$$

Applying equations (4.1) and (4.3) in (4.2) we have:

$$\lambda = \frac{\upsilon_1 t}{|a|} \qquad (26)$$

where, $|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

So, equation (23) now can be rewritten with respect to equation (26) as follows:

$$\begin{cases} x = x_1 + (\dfrac{a_1\upsilon_1}{|a|})t \\ y = y_1 + (\dfrac{a_2\upsilon_1}{|a|})t \\ z = z_1 + (\dfrac{a_3\upsilon_1}{|a|})t \end{cases} \qquad (27)$$

Hence, each time instant we know at exactly which point of a straight line the aircraft 1 is located. We work similarly for all others. If the distance between the centers of the safety spheres of the aircrafts is smaller than the sum of their radius at time t, this means that the aircrafts collide. Thus, by choosing equal small time intervals we detect during each one whether a collision occurs.

B. Case Study: Three Randomly Distributed Aircrafts

In the following simulations all aircrafts are assumed crossing the control volume with the same speed. The control volume has a radius of 67 nm or 108 km and the minimum safety distance has been set to 5.6 nm or 9 km. Two plots are presented for each case study, one that shows the aircraft configuration and their projected trajectories before maneuvers are made to avoid possible conflicts and the next shows the corresponding situation after the various speed maneuvers. Each case study is accompanied by a table that shows the velocities of the aircrafts before and after the conflict resolution in order to compare the various cases.

In Figure 7 we see three randomly distributed aircrafts which are all headed inside the control volume. The final configuration points of them are also presented. In this case study, there is one conflict between aircrafts 1 and 2 which is resolved by velocity maneuvers of all aircrafts 1 and 2 and 3; all interacting aircrafts cooperate towards optimization of a common goal, as agents in the same team.

In Figure 4.3 we see the aircrafts and their trajectories after the maneuvers for conflict resolution. It is important to remember that an aircraft does not change its trajectory in order to avoid a conflict; it just merely changes its speed (accelerate or decelerate). This means that in all the following plots, identical trajectories between two consecutive plots do not imply absence of maneuver in general, but rather absence of heading angle change. In the specific example that we study, the conflicts were resolved only by velocity changes. The values of the velocities of the aircrafts before and after the conflict resolution are shown in Table 1.

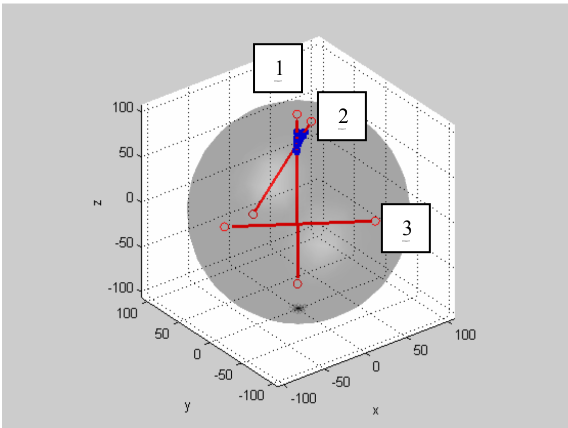

Figure 7. Three randomly distributed aircrafts and their projected trajectories before conflict resolution. With blue color we present the area where the conflict is detected.

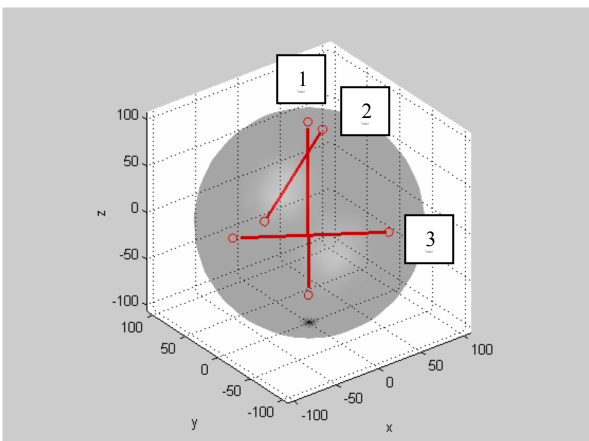

Figure 8. Three randomly distributed aircrafts and their projected trajectories after conflict resolution. No conflict occurs after the velocity changes.

We notice that cases like head-to-head conflicts can be easily solved with a heading angle change maneuver, but not only with velocity change maneuver. Also conflicts that occur in time near $t = 0$ can not be solved always by the velocity change problem because the upper and lower bounds on the velocity υ_i, are not adequate in order to avoid the conflict.

All the changes in velocities after conflict resolution are optimal with respect to the common goal of aircrafts. This means, that for aircraft 1 the velocity change q_1=-0.703 is the minimum acceptable deceleration that could be applied in order to avoid conflict with aircraft 2.

	Aircraft	**Aircraft**	**Aircraft**

	1	**2**	**3**
Velocity before C.R. (km/min)	15	15	15
Change in Velocity after C.R. (km/min)	-0.703	0.99	0.99

Table 1. Table showing the velocities for three randomly distributed aircraft. CR stands for Conflict Resolution.

IV. Conclusions

Several conflict resolution maneuvers have been considered and one relative model has been presented (VC).

Based on geometric construction of the conflict avoidance constraints a linear minimization problem with non linear constraints and together with integer variables has been obtained. The CPLEX software package has been used to solve the problem and due to the fast computation of the tool optimal solutions have been found quickly (in very few seconds).

Future investigations of the optimal maneuvers (velocity change and heading angle) in the three-dimensional space, in terms of flight time, are part of future work. Due to the non linearity that follows from considering heading angle and velocity variation, a future work is to consider other variables and formulate the problem as mixed integer programming. In another direction, more case studies should be examined, with more complex configuration patterns and a greater number of aircrafts so as to gain more insight in the algorithm and the way it works, although the situations examined here are realistic.

References

[1] Air Traffic Control Website: http://atcsl.tripod.com/air_traffic_control_history.htm .

[2] J.K. Kuchar and L.C. Yang, “A Review of Conflict Detection and Resolution Modeling Methods”, *IEEE Transactions on Intelligent Transportation Systems*, vol. 1, No. 4, pp. 179-189, December 2000.

[3] K.D. Bilimoria, and H.Q. Lee, “Properties of Air Traffic Conflicts for Free and Structured Routing”, *in proc. AIAA Guidance, Navigation and Control Conference*, August 2001, paper No 2001-4051.

[4] J.-H. Oh and E. Feron “Fast detection and resolution of multiple conflicts for 3-Dimensional free flight”. *in proc. 36th IEEE Conference on Decision and Control*, Dec. 1997.

[5] *ILOG CPLEX Users Guide.* ILOG, 1999.

[6] L. Pallottino, E.M. Feron and A. Bicchi, “Conflict Resolution Problems for Air Traffic Management Systems Solved with Mixed Integer Programming”, *IEEE Transactions on Intelligent Transportation Systems*, Vol. 3 , No. 1 , pp. 3-11, March 2002.

[7] F.S. Hillier, G.J. Lieberman, *Introduction to operations research, 6th edition.* New York: McGraw Hill, 1995.

[8] C.A. Floudas. *Non-Linear and Mixed-Integer Optimization*. London: Oxford University Press,1995.