Decentralized load-frequency control of a two-area power system via linear programming and optimization techniques

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***Abstract*-In this paper we consider the decentralized load-frequency control (LFC) of a two-area power system. The model of the power system is determined, and is viewed as an interconnected system (A, B_d), for which the methods of the decentralized control are applied; these are based on the intercontrollability matrix D(s), its kernel U(s), and the equivalent system {M(s), I_2} in the operator domain. Based on this, a decentralized control via linear programming methods, and with decentralized optimization techniques is determined. Several responses are given and compared.**

***Index Terms*- Decentralized control, Load-frequency control, Linear programming, Optimization techniques, Static local feedback.**

I. INTRODUCTION

The problem of designing a feedback gain matrix for load-frequency control in electrical power systems has received considerable attention. Several decentralized load-frequency controllers have been developed since the 1970s ([5],[6], [7], [11], [12], [17], [20]).

In this paper, we propose a decentralized LFC controller based on the stabilization problem with static feedbacks of the local state vectors in an interconnected power system. It is examined for an interconnected (global) system (A, B_d), consisting of two local scalar subsystems, under the very general assumptions of the global and the local controllability. Only the case of two interconnected subsystems is examined, since only in this case the global system will have no decentralized fixed modes when local feedbacks are applied ([18], [4], [1], [2], [9], [10]). Without loss of generality ([4], [18]), we assume that both input channels of system (A, B_d) are scalar.

In the next section we present some preliminary results needed in the main development. They concern the form of matrices A and B_d, the intercontrollability matrix D(s) and its kernel U(s), and -based on this- an equivalent, to (A, B_d), system, defined in the operator domain. The existence of linear, local, state-vector feedbacks (LLSVF), which stabilize the system, is formally proven in section III. In this context, and based on linear programming methods, their computation is presented ([15]). Section IV provides an algorithm proposed by Geromel and Bernussou, which based on iterative scheme gives a solution to the near-optimum decentralized control problem. In section V, first the model of the system is analytically developed. Then, the system is stabilized with the proposed method, based on linear programming, which gives also an admissible stabilizing matrix for the traditional method of the decentralized optimization techniques, as given in the works of Geromel and Bernussou ([12]). Several responses are given and compared.

II. PRELIMINARIES

A. Form of matrices A and B_d

We consider the interconnected system (A, B_d) defined by

$$\dot{x} = Ax + B_d u \tag{1}$$

where x is the n-dimensional state vector of (A, B_d), u is the 2-dimensional input vector, A is the nxn system matrix, and B_d is its nx2 input matrix. Matrices A and B_d admits the following partitioning:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{matrix} \updownarrow n_1 \\ \updownarrow n_2 \end{matrix} \quad \text{and} \quad B_d = \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} \tag{2}$$

with $n=n_1+n_2$. System (A, B_d) consists of the interconnected n_i- dimensional subsystems (A_{ii}, b_{ii}) -i=1,2- of local state vectors $x_1$and x_2, with $x=[x_1' \; x_2']'$, u_1 and u_2 being, respectively, the scalar inputs of these subsystems, with $u=[u_1 \; u_2]'$. We further assume that the global system (A, B_d), as well as its two subsystems (A_{ii},b_{ii}) -i=1,2- are controllable. In that case, subsystems (A_{ii}, b_{ii}) are supposed to be in their companion controllable form [13]. It is obvious that when the various submatrices of (A, B_d) are in the above form, system (A, B_d) is called Canonical Interconnected Form, CIF [8]. Finally, with the elements of rows n_1 and n_1+n_2 (=n) of A, we form matrix A_m:

$$A_m = \begin{bmatrix} a'_{11} & a'_{12} \\ a'_{21} & a'_{22} \end{bmatrix} \tag{3}$$

B. *The intercontrollability matrix D(s) and its kernel*

The following (n-2)xn polynomial matrix is the intercontrollability matrix of system (A,B_d)

$$D(s)=\left[\begin{array}{cccc|cccc} s & -1 & & & & & & \\ & \cdot & \cdot & & & -A_{12}^{0} & & \\ & & \cdot & \cdot & & & & \\ & & s & -1 & & & & \\ \hline & & & & s & -1 & & \\ & -A_{21}^{0} & & & & \cdot & \cdot & \\ & & & & & & \cdot & \cdot \\ & & & & & & s & -1 \end{array}\right] \quad (4)$$

As the following lemma indicates, D(s) expresses the conditions for the controllability of (A, B_d):

Lemma 1 *System (A, B_d) is controllable if and only if rank D(s) = n-2 for all complex numbers s.*

For the proof, see [2].

Thus, the matrix D(s) of a controllable system is a full-rank matrix. Its kernel U(s) is an nx2 polynomial matrix of rank 2, such that D(s) U(s) = 0. The analytical determination of U(s) is as follows: P is the matrix representing the column permutations of matrix D(s), which brings it to the form of the matrix pencil:

$$\tilde{D}(s) = D(s)\, P = [s\, I_{n-2} - F \mid G] \quad (5)$$

In (5) G is an (n-2)x2 (constant) matrix, consisting of columns n_1 and $n_1+n_2=n$ of D(s), F is an (n-2)x(n-2) constant matrix, and I_{n-2} is the unity matrix of order n-2. Since D(s) is a full rank matrix, the pair (F, G) is controllable, and can be brought to its multivariable controllable form (MCF) [13], [19] $(\hat{F},\hat{G})$ by a similarity transformation T; let d_1, d_2 be the controllability indices of (F, G), S(s) be the associated structure operator, δ(s) be the characteristic (polynomial) matrix of $\hat{F}$, and (in case rank[G]=2) let $\hat{G}_m$ be the 2x2 matrix consisting of rows d_1 and $d_1+d_2=n-2$ of $\hat{G}$. The precise form of U(s) is the content of the following lemma:

Lemma 2 *Let D(s) be the intercontrollability matrix of (A, B_d) as in lemma (1), and suppose that rank[G]=2, for G as in (5). Then the kernel U(s) of D(s) is equal to*

$$U(s) = P\begin{bmatrix} TS(s) \\ \hdashline -\hat{G}_m^{-1}\delta(s) \end{bmatrix} \quad (6)$$

where P, T, S(s), $\hat{G}_m$, and δ(s) are as previously explained.

C. *An equivalent system defined by a PMD.*

Consider the interconnected system (A, B_d), with A and B_d as in (2). In that case, the corresponding differential equation in the state space is:

$$\dot{x}(t) = Ax(t) + B_d u(t) \quad (7)$$

In the operator domain, this equation corresponds to the equation $\quad (sI - A)\, x(s) = B_d\, u(s) \quad (8)$

this, in its turn, is reduced to the equations:

$$D(s)\, x(s) = 0 \quad (9a)$$

$$(sE - A_m)\, x(s) = u(s) \quad (9b)$$

In these equations, D(s) is as in (4), E is a 2xn (constant) matrix, of the form:

E =diag$\{e_1{}'\ e_2{}'\}$, the n_i-dimensional vector e_i being equal to : $e_i = [0...0\ 1]'$ -for i=1,2-, and A_m is the matrix defined in (3). From (9a) it follows that x(s) must satisfy the relation:

$$x(s) = U(s)\, \xi(s) \quad (10)$$

where U(s) is the kernel of D(s), and ξ(s) is any two-dimensional vector. It follows that ξ(s) must satisfy the equation: $\quad M(s)\, \xi(s) = u(s) \quad (11)$

The matrix M(s) appearing in (11) is termed characteristic matrix of the interconnected system (A, B_d) [8], and is defined by the relation:

$$M(s) = (sE - A_m)\, U(s) \quad (12)$$

The three systems defined respectively (i) in the state space by the pair of matrices (A, B_d), (ii) in the operator domain by $\{sI\text{-}A, B_d\}$, and (iii) by the polynomial matrix description (PMD):

$$M(D)\, \xi(t) = u(t) \quad (13a)$$

$$x(t) = U(D)\, \xi(t) \quad (13b)$$

are equivalents [13], [9], [3]. It is noted that in (13) ξ(t) is the pseudo state vector of the system, and is related to the state vector x(t) of (A, B_d), by the relation

$$x(t) = U(D)\, \xi(t) \quad (14)$$

(in the relations (13), (14), the symbol D denotes the differential operator d/dt).

III. COMPUTATION OF THE LOCAL FEEDBACK STABILIZING MATRIX K_d VIA LINEAR PROGRAMMING METHODS.

We first present a result, as lemma (3), which will be needed in the proof of the Main Theorem (3).

Lemma 3. *Let h(s) be a polynomial of the form: h(s)=r(s)p(s)+q(s), for which the following assumptions hold: (i) The polynomials r(s), p(s), q(s) are monic (ii) r(s) is arbitrary, (iii) degree r(s)p(s) > degree q(s) (iv) p(s) is a stable polynomial. Then, the arbitrary polynomial r(s) can be chosen so, that h(s) is stable.*

For the proof, see [16].

Theorem 3 *Consider the interconnected system (A, B_d) as in (1), and suppose that the global system (A, B_d), and the local ones (A_{ii}, b_{ii}) -i=1,2- are controllable. Then, there exists static LLSVF of the form $u=K_d x$, so that the resulting closed-loop system is stable.*

Proof. For the proof we consider the equivalent system $\{M(s), I_2\}$ and examine the stability of the polynomial matrix $M_d(s)=(sE-A_m-K_d)U(s)$, by examine whether its determinant is a stable polynomial. We assume that the feedback matrix K_d has the form:

$$K_d = \left[\begin{array}{ccc|ccc} \alpha_1 & . \; . & \alpha_{n_1} & 0 & . \; . & 0 \\ \hline 0 & . \; . & 0 & \beta_1 & . \; . & \beta_{n_2} \end{array}\right] = \begin{bmatrix} \alpha' & \mathbf{0} \\ \mathbf{0} & \beta' \end{bmatrix} \quad (15)$$

where α_i ($i=1,...,n_1$), and β_j ($j=1,...,n_2$) are some unknown, real numbers. We shall deal with the case where rank[G]=2, which is the usual one for the matrix G. Without loss of generality, and in order to simplify the notation, we assume $A_m=0$ (see also Remark 3.1 after the end of the proof). Then the matrix $M_d(s)$ takes the form

$$M_d(s)=(sE-A_m-K_d)U(s)=(sE-A_m-K_d)\,P\left[\begin{array}{c} TS(s) \\ \hline -\hat{G}_m^{-1}\delta(s) \end{array}\right]=$$

$$=\left[\begin{array}{c|c} -(s-\alpha_{n1})[11]-\alpha(s) & -(s-\alpha_{n1})[12]-\alpha_1(s) \\ \hline -(s-\beta_{n2})[21]-\beta_1(s) & -(s-\beta_{n2})[22]-\beta(s) \end{array}\right]$$

$$=\begin{bmatrix} M_{11}(s) & M_{12}(s) \\ M_{21}(s) & M_{22}(s) \end{bmatrix} \quad (16)$$

where $\alpha(s) = [\ \alpha_1 ... \alpha_{n_1-1}\ 0\ 0\ ]\ TS_1(s)$

$\alpha_1(s) = [\ \alpha_1 ... \alpha_{n_1-1}\ 0\ 0\ ]\ TS_2(s)$

$\beta(s) = [\ 0\ ...\ 0\ \beta_1 ... \beta_{n_2-1}\ ]\ TS_2(s)$

$\beta_1(s) = [\ 0\ ...\ 0\ \beta_1 ... \beta_{n_2-1}\ ]\ TS_1(s)$

are scalar polynomials, not monic,

$TS_1(s) = T\ [\ 1\ s\ ...\ s^{d_1-1}\ 0\ ...\ 0\ ]'$

$TS_2(s) = T\ [\ 0\ ...\ 0\ 1\ s\ ...\ s^{d_2-1}\ ]'$

(i.e., $TS(s)=TS_1(s)\ TS_2(s)]$,and [ij] -for i,j=1,2- are the entries of the polynomial matrix $\hat{G}_m^{-1}\delta(s)$. Then the matrix in (16) is equivalent to the following matrix:

$$\begin{bmatrix} M_{11}(s)+M_{21}(s) & M_{12}(s)+M_{22}(s) \\ M_{21}(s) & M_{22}(s) \end{bmatrix}$$

The determinant of this matrix is actually a monic polynomial of the form $h(s)=r(s)p(s)+q(s)$, of degree n, by identifying r(s) as the polynomial $[22][M_{11}(s)+M_{21}(s)]$, which is of degree (n-1), arbitrary and monic, p(s) as the polynomial $(s-\beta_{n_2})$, which is stable by choice of β_{n_2}, and q(s) as the polynomial:

$$[M_{12}(s)+M_{22}(s)]M_{21}(s)-[M_{11}(s)+M_{21}(s)]\beta(s)$$

of degree (n-1). Then, according to lemma (3), the arbitrary polynomial r(s) can be chosen so that the polynomial h(s) is stable. Q.E.D.

Next, we propose an iterative method, in order to compute the feedback coefficients [15]. The central idea is to compute the feedback parameters by solving a linear programming problem, corresponding to choosing positive the coefficients of the polynomials that should be stable. A set of such polynomials (with positive coefficients) is generated. They are then examined whether they are stable or not.

ALGORITHM

Step1 Choose the feedback parameter β_{n_2}, so that a stable p(s) results.

Step2 Write the polynomial r(s) in the following form:

$r(s) = s^{n-1}+k\rho(s)=s^{n-1}+k(s^{n-2}+k_1 s^{n-3}+...+k_{n-2})$.

By viewing the degrees of the polynomials $\alpha(s)$ and $\beta(s)$, it is seen that k -the leading coefficient of the polynomial $\rho(s)$- contains only the parameters β_{n_2} and α_{n_1}. It follows that by giving a value to k, we can also compute α_{n_1}.

Step3 Form n-2 inequalities with the n-2 unknown feedback parameters, by setting positive the coefficients k_i of the polynomial $\rho(s)$ ($k_i>0$, for i=1, n-2).

Step4 Solve the linear programming problem , by putting an objective function with unity weighting coefficients, and find all feedback parameters α_i ($i=1, n_1-1$) and β_j ($j=1, n_2-1$).

Step5 Evaluate the polynomial $\rho(s)$, and check if it is stable. If it is not, go back to Step 1, and select another β_{n_2}.

Step6 Evaluate the polynomial r(s), and check if it is stable. If it is not, go back to Step 2, and select another k.

Step7 Evaluate the polynomial h(s), and check if it is stable. If it is not, go back to Step 2, and select another k.

Step8 The feedback matrix K_d can be evaluated from steps 1, 2 and 4.

END OF THE ALGORITHM

REMARKS

Remark 3.1 : When $A_m \neq 0$, the proof follows exactly the same lines, since the orders of the various polynomials remain the same. Now, however, the notation is more complicated, since the elements of A_m appear in the polynomials $\alpha(s)$, $\beta(s)$, and, additionally, multiply the matrix TS(s).

Remark 3.2: This iterative method probably will give large parameters of the feedback matrix K_d. These parameters can be changed, by applying an optimization algorithm, as the one described in paragraph IV.

Remark 3.3: The linear programming problem was solved via the revised simplex algorithm [14], which is contained in the DLPRS subroutine of the IMSL/MATH Library.

IV. DECENTRALIZED CONTROL BY PARAMETRIC OPTIMIZATION

This section provides an algorithm [12], based on iterative scheme, for designing 'optimal' decentralized control. It is to be emphasized that the algorithm needs to be initialized with a stabilizing control, which is the feedback matrix K_d of the proposed previously method based on linear programming.

The problem is to find a feedback matrix K_d such that the global system as (2) is stable and that the classical quadratic cost function $C=\int(x'Qx+u'Ru)dt$ is minimized, where Q and R are weighting matrices of appropriate dimensions, positive-semi definite and positive define, respectively.

Geromel and Berussou [12] proposed a gradient method which is summarized below:

Step1 Determine the gradient matrix $\partial C(K_d)/\partial K_d$ and the feasible direction D=diag$\{D_1, D_2\}$.

Step2 Test of convergence -if $|\{d_{pq}\}_i|<\varepsilon$ for all p=1; q=1,...,n_i; i=1, 2; where $\{d_{pq}\}_i$ is the (p, q) entry of the matrix D_i, stop, if not go to Step 3.

Step3 Update on K_d ($K_d \leftarrow K_d-\alpha D$), where the step size $\alpha\geq 0$ must selected such that $C(K_d-\alpha D)<C(K_d)$ and go back to Step 1.

V. A TWO-AREA ELECTRICAL POWER SYSTEM

We consider the P-f control of a two-area power system. The system model was derived in ([6], [11]), and its block-diagram is given in Fig. 1 (disregard, for the time being, the portions of the diagram marked with dotted lines). The model is based on the equations for the power equilibrium, the equations for the incremental tie-line flow, the equations for the change in generation, and the equations for the position of the speed governor. These equations are as follows:

$$\frac{2H_i}{f^0}\frac{d\Delta f_i}{dt} + D_i\Delta f_i + 2\pi\sum_{j\neq 1} T_{ij}^0\left(\int \Delta f_i dt - \int \Delta f_j dt\right) = \Delta P_{gi} - \Delta P_{Di} \tag{17}$$

$$\Delta P_{tiei} = \int 2\pi T_{12}^0(\Delta f_i - \Delta f_j)dt, i\neq j; i,j=1,2 \tag{18}$$

$$\frac{d\Delta P_{gi}}{dt} = -\frac{1}{T_{ti}}\Delta P_{gi} + \frac{1}{T_{ti}}\Delta x_{Ei} \tag{19}$$

$$\frac{d\Delta x_{Ei}}{dt} = -\frac{1}{T_{gi}}\Delta x_{Ei} + \frac{1}{T_{gi}R_i}\Delta f_i + \frac{1}{T_{gi}}\Delta P_{ci} \tag{20}$$

The meaning, and the values, of the various coefficients of (17-20) can be found in the Appendix, at the end of the paper.

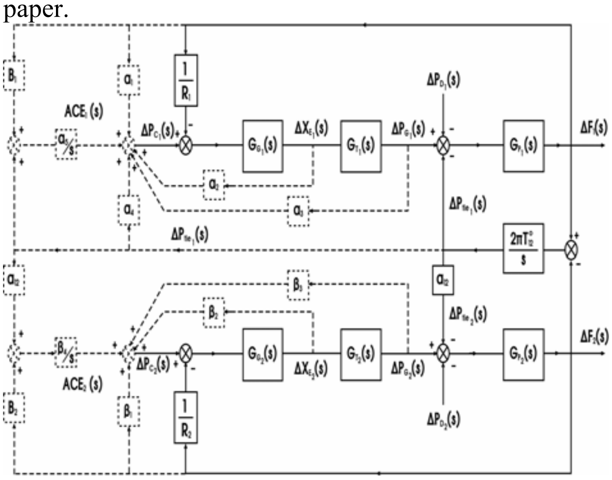

Fig. 1. A two-area power system

For the two area system, the states and the control variables are chosen as:

$$x' = \left[\Delta f_1 \Delta x_{E1} \Delta P_{g1} \Delta P_{tie1} \Delta f_2 \Delta x_{E2} \Delta P_{g2}\right] \tag{21}$$

$$u' = \left[\Delta P_{c1} \Delta P_{c2}\right] \tag{22}$$

The basic objectives of the P-f control are zero steady-state error in the deviation of the frequency and the tie-line power. In order to achieve them, it is essential to consider an integral of the area control error (ACE = BΔf+ΔPtie) as a feedback signal. The state vector in (21) is augmented with two additional state variables, which are defined by: $x_5 = \int ACE_1 dt$, $x_9 = \int ACE_2 dt$

and which satisfy the equations:

$$\dot{x}_5 = ACE_1 = B\Delta f_1 + \Delta P_{tie1}$$

$$\dot{x}_9 = ACE_2 = B\Delta f_2 + \Delta P_{tie2} = B\Delta f_2 + a_{12}\Delta P_{tie1}$$

$$\Delta P_{tie2} = a_{12}\Delta P_{tie1},\quad a_{12} = -\frac{P_{r1}}{P_{r2}}$$

We substitute the definition of the states and the controls into the nine differential equations that define the two-area system. This place the system equations into the form: $\dot{x} = Ax + B_d u + \Gamma_d \Delta P_D$. The matrices A, B_d and Γ_d of dimensions 9x9, 9x2 and 9x2 respectively.

$$A = \left[\begin{array}{ccccc|cccc}
-\frac{f^0 D_1}{2H_1} & 0 & \frac{f^0}{2H_1} & -\frac{f^0}{2H_1} & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{T_{g_1}R_1} & -\frac{1}{T_{g_1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{T_{t_1}} & -\frac{1}{T_{t_1}} & 0 & 0 & 0 & 0 & 0 & 0 \\
T_{12}^* & 0 & 0 & 0 & 0 & -T_{12}^* & 0 & 0 & 0 \\
B_1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & -\frac{a_{12}f^0}{2H_2} & 0 & -\frac{f^0 D_2}{2H_2} & 0 & \frac{f^0}{2H_2} & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{g_2}R_2} & -\frac{1}{T_{g_2}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{t_2}} & -\frac{1}{T_{t_2}} & 0 \\
0 & 0 & 0 & a_{12} & 0 & B_2 & 0 & 0 & 0
\end{array}\right]$$

$$B_d' = \left[\begin{array}{ccccc|cccc}
0 & \frac{1}{T_{g_1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{T_{g_2}} & 0 & 0
\end{array}\right]$$

$$\Gamma_d' = \left[\begin{array}{ccccc|cccc}
-\frac{f^0}{2H_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & -\frac{f^0}{2H_2} & 0 & 0 & 0
\end{array}\right] \tag{23}$$

The matrix Γ_d is the disturbance distribution matrix, whereas the vector

$$\Delta P_D = \left[\Delta P_{D1} \quad \Delta P_{D2}\right] \tag{24}$$

is the disturbance vector. We redefine the states in terms of their steady-state values, i.e. $x_i^* = x_i - x_{iss}$ for i=1...9.

This change of variables puts the system into the form $\dot{x}^* = Ax^* + B_d u$ with $x^*(0) = -x_{ss}$ The matrices A and B_d remain unchanged.

In order to prevent unnecessary complications in the notation, we drop the superscript (*) in the sequel.

The eigenvalues of A as in (23) (for the numerical values which are given at Appendix) are: -13.29; -13.26; -1.62; -1.3±j2.5; -0.5±j3.5; 0; 0. The system is unstable, since it has two zero eigenvalues, due to the integrators of the secondary control loops. In order to compute a stabilizing K_d matrix as in (15), we apply the linear programming method. The static feedbacks are shown in Fig. 1, in the portion of the figure marked with dotted lines, with $k_{1i}=\alpha_i$ for i=1...5, and $k_{2i}=\beta_i$ for i=1...4. The method described in paragraph III results into the following values:

$$CIFK_d = \left[\begin{array}{ccccc|cccc} -2.5 & -2.5 & -2.5 & -2.5 & 3.9 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & -1.93 & -2.5 & -0.17 & 11.9 \end{array}\right]$$

It is remarked that the above values of $CIFK_d$ are in the transformed system of coordinates (used to apply the method based on the linear programming), whereas in the initial system of coordinates $CIFK_d$ has the values:

$$K_d = \left[\begin{array}{ccccc|cccc} -0.06 & 0.31 & -0.38 & 0.06 & -0.02 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & -0.01 & 0.95 & -0.97 & -0.02 \end{array}\right]$$

The system is now stable, since its eigenvalues (i.e. the eigenvalues of $A+B_dK_d$) are: -8.3; -2.13±j1.35; -1.23±j3.71; -0.48±j6.25; -0.018; -0.017. The responses of the frequencies Δf_1 and Δf_2, of the change in the tie-lie power ΔP_{tie} and of the generated powers ΔP_{g1} and ΔP_{g2}, to a 1% increase of the load, are shown in the Fig. 2

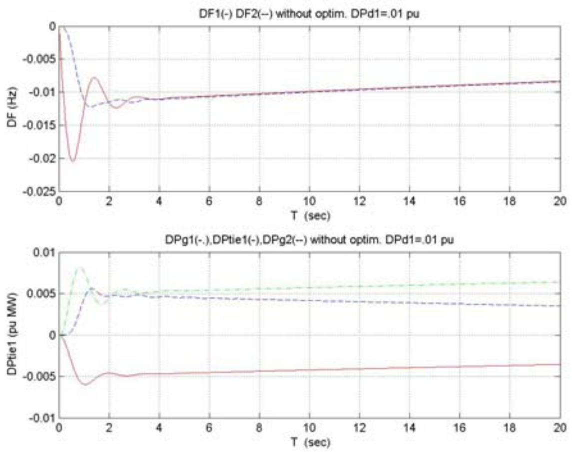

Fig. 2. Frequency responses $\Delta f_1(t)$, $\Delta f_2(t)$ and the generated powers $\Delta P_{g1}(t)$ and $\Delta P_{g2}(t)$ of areas 1 and 2 to a load increase ΔP_{D1}=0.01 pu, with the decentralized stabilizing feedback K_d.

The settling time of these responses is out of the specifications. So, in step 3 of the proposed algorithm, by changing the constraints limits of the revised simplex method, we achieve a new feedback matrix K_d (referred in the initial system of coordinates):

$$K_d^n = \left[\begin{array}{ccccc|cccc} -0.47 & 1.03 & -2.97 & 1.8 & -0.59 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & -0.31 & -3.85 & 3.26 & -0.59 \end{array}\right]$$

The eigenvalues of closed-loop system, (i.e. the eigenvalues of $A+B_dK_d$) are now: -62.95; -1.03; -0.35; -0.6±j0.45; -0.54±j11.03; -0.20±j2.51. The frequency and the generated power responses, to a load change of 1%, are shown in Fig. 3, which satisfy all the static and dynamic specifications.

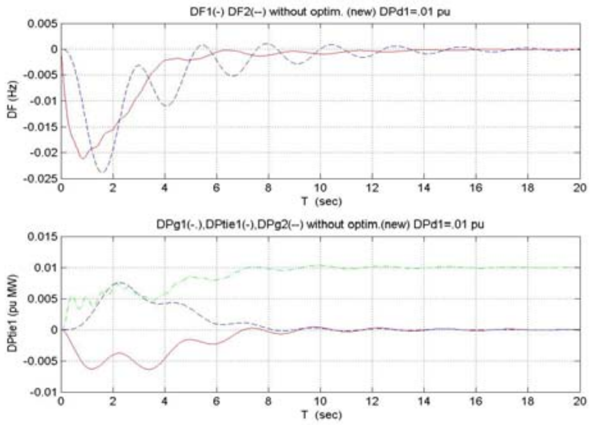

Fig.3. Frequency responses $\Delta f_1(t)$, $\Delta f_2(t)$ and the generated powers $\Delta P_{g1}(t)$ and $\Delta P_{g2}(t)$of areas 1 and 2 to a load increase ΔP_{D1}=0.01 pu, with the new decentralized stabilizing feedback K_d^n .

In case the feedback elements of K_d are considered to be large or the dynamic and static behavior is not so ''good'', we make use of the decentralized optimization algorithm as in paragraph IV.

We define the Q and R matrices, necessary in the optimization algorithm [12], by defining a set of requirements which the system should satisfy:

1. The static frequency deviation Δf_{istat} and the static change in the tie-line power ΔP_{tie}, following a step load change, must be zero.
2. The transient frequency deviation should not exceed ±0.02Hz under normal conditions, and the time error should not exceed ± 3 seconds.

Expressing all this in mathematical form, we must have the sum of following terms:

$$(\Delta f_1)^2 + (\Delta f_2)^2 + (\Delta P_{tie_1})^2 + \left(\int ACE_1 dt\right)^2 + \left(\int ACE_2 dt\right)^2$$

This sum is put in the form x'Qx. Large control efforts are penalized by adding the terms: $(\Delta P_{c1})^2 + (\Delta P_{c2})^2$. This requires that the matrix R -corresponding to the term u'Ru- is the 2x2 unity matrix. The cost function is defined as $C = \int (x'Qx + u'Ru)dt$

Since $A_f=A+B_dK_d$ is stable, we apply the decentralized optimization technique [12] by selecting properly the entries of Q and R matrices for achieving ''better'' responses. This results in an optimal $CIFK_d^{opt}$ matrix, with the values in the original system of coordinates equals to:

$$K_d^{opt} = \left[\begin{array}{ccccc|cccc} -0.03 & 0.29 & -1.33 & 1.31 & -0.47 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & -0.01 & 0.39 & -0.45 & -0.6 \end{array}\right]$$

The eigenvalues of $A+B_dK_d^{opt}$ are now: -6.31; -5.08±j6.32; -1.86; -1.10 ±j3.31, -0.99±j0.45; -0.73. The frequency and the generated power responses, to a load change of 1%, are shown in Fig. 4.

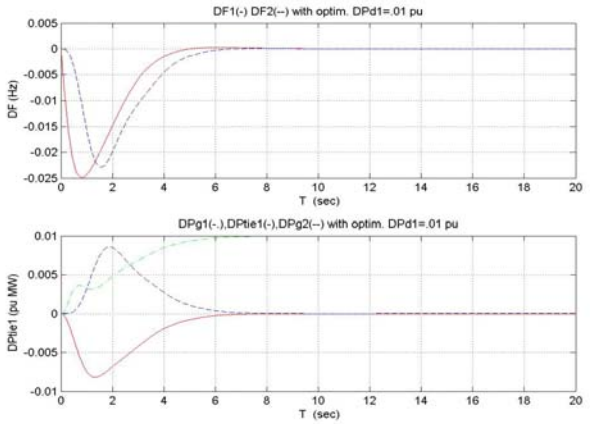

Fig. 4. Frequency responses $\Delta f_1(t)$, $\Delta f_2(t)$ and the generated powers $\Delta P_{g1}(t)$ and $\Delta P_{g2}(t)$ of areas 1 and 2 to a load increase ΔP_{D1}=0.01 pu, with the optimal decentralized stabilizing feedback K_d^{opt}.

From a comparison of the responses as in Fig. 3 and 4 it seems that the responses in the last are "better" than in the other, since they have less oscillatory behavior and settling times. It should be also noted that in the last (where the optimal decentralized feedback is applied to the system), the feedback coefficients are indeed smaller i.e. easier to realization.

VI. CONCLUSION

In this paper we considered the decentralized control of a power system, resulting from the interconnection of two local power systems. After the system model was determined, it was viewed as an interconnected system (A, B_d) for which the methods of the decentralized control of interconnected systems were applied (intercontrolability matrix D(s), its kernel U(s), equivalent system {M(s), I_2} in the operator domain, stabilization with local state feedbacks via linear programming methods and optimization techniques). Several responses of the simulated system were finally determined and analyzed.

APPENDIX

The two identical control areas have the following system data (the method is also applicable to systems that are not identical):
Nominal frequency: f^0= 60 Hz
Total rated area capacity: Pr=2000 MW
Nominal operating load: P_D^0=1000 MW
Inertia constant: H=5.0 s
Regulation (4 percent drop in speed between no load, and full load): R=2.4 Hz/pu MW
Load constant (1 percent increase in load, for 1 percent frequency increase): D=8.33 x10pu MW/Hz
Time constant of a non-reheat turbine generator: T_t= 0.3s
Time constant of speed-governing mechanism: T_g= 0.08s
Tie-line capacity: P_{max12}= 200 MW
Nominal tie-line power angle: $\delta_1^0 - \delta_2^0 = 30^0$
Synchronizing coefficient: $T_{12}^* = 2\pi T_{12}^0$
$T_{12}^0 = P_{12}^0 \cos(\delta_1^0 - \delta_2^0) = 0.545 / 2\pi = 0.087$,
Frequency bias parameter: B=0.425 pu MW/Hz
Load disturbance parameter: ΔP_D= 0.01 pu MW.

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