Controlling the Loop-Gain for Robust Adaptive Control of a Mechatronic System

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***Abstract*— This paper concerns the question of applicability of adaptive control strategies in real environments. Because of unrobustness to unmodeled dynamics – especially dead time – model reference adaptive control with all its positive features can not be implemented in industry. But it can be shown that an additional gain-controller within the MRAC-concept leads to a robust adaptive controller applicable to real systems. In this context, the paper gives a possibility of closing the gap between theory and praxis in the field of adaptive control. As a case study, a two-mass flexible servo system with unknown inertia, spring and damping constants is investigated while the dynamics of the power converter, speed-sensor and further unknown and time-varying dead-times can be neglected. The goal is a perfect dynamic tracking of the load-mass speed with a smooth control output.**

***Index Terms*— Model Reference Adaptive Control, Unmodeled Dynamics, Robustness, Gain-Control, Flexible Servo**

I. Introduction

In classical control theory, complete knowledge of a system is necessary to design a stable controller with good performance. But in most industrial applications the designer is confronted with a complex plant, e.g. continuous processing plants with coupled servo drives, and thus he has just a rough idea about the system to be controlled. According to this, the plant has to be identified – next problems arise: you never know if the real parameters were found and if they are drifting with time. Based on these uncertainties the controller will be designed with the goal to be stable and to show good tracking behavior. At this point one can recognize the problem of a serial sequence of identification and control. Up to now the negative effects in control were minimized by a conservative controller design but in these days a compromise between quality and quantity, i.e. production-speed, is no longer acceptable. High-dynamic processing is the keyword.

A perfect theoretical method to cope with the described problems is to make the controller adaptive. If all the uncertainties are parametric, their effect can be completely eliminated by adjusting the parameters of an underlying identification model – in contrast to the described classical theory identification and control take place parallel. Adaptive control is a well established discipline and proofs of stability as well as conditions for parameter convergence are available for continuous [1] and discrete systems [4]. Because every controller is digital in these days we will concentrate in the following on the discrete form of model reference adaptive control (MRAC).

II. Statement of the Problem

In theory, with MRAC, every unknown system can be controlled with success – perfect dynamic behavior with guaranty of stability. The "only" prior information needed is order and relative degree of the plant. This information is directly connected to the number of parameters, that have to be identified to describe the system. At this point the main problem arises that makes application of MRAC in practice impossible: there is no model which perfectly describes a system as well as the environment in which the system operates. Every physical system is of arbitrary order. In fact, for the design of the controller, the order of the model should be as low as possible to keep simplicity. For that, the dominant dynamics, i.e. the main system-behavior, is separated from the parasitic dynamics. In linear control theory by the choice of the reference signal/desired value excitation of unmodeled parasitic dynamics can be avoided, or in other words, if the designer is aware of the requirements the dominant order is known and unmodeled high-frequency dynamics result in no negative effects. But in adaptive control theory things look different. Because of the time varying parameters, the controller is nonlinear – the excitation of the system is no longer only a result of the desired value but also a result of the dynamics of the nonlinear adaptation. At the beginning of an adaptive control process wrong parameters result in large control outputs of high frequency. If there is no unmodeled dynamics it ends up after short time in a perfect tracking and smooth control output according to adaptive control theory. But if there is unmodeled dynamics it will be excited by the high frequency of the control output. In consequence the underlaying identification can not find an appropriate constant set of parameters that represents the system behavior. Thus parameters keep varying and lead again to a control output exciting the parasitic dynamics – it is a vicious circle that results in instability.

Now, it is clear that MRAC is not applicable to real systems. The problem is to make the adaptive controller robust to the remaining uncertainties that arise from unmodeled dynamics. It was already pointed out by Rohrs et.al. [2]

that adaptive systems are highly non-robust to unmodeled dynamics.

In the present paper in section IV, a modification of the adaptive MRAC-concept is presented to yield a robust adaptive controller applicable to real scenarios. The main idea is to reduce the loop-gain if unmodeled dynamics are excited such that parasitic dynamics will not be excited any more – the above vicious circle can be broken. This is comparable to the aim of suiting the controller dynamics to the dynamics represented by the chosen order of the model. With such a "filtered" gain-controlled input the error due to unmodeled dynamics is almost zero and the existing proof of stability can be expected to proceed as in the ideal case where all the uncertainties are parametric.

III. Unrobust Model Reference Adaptive Control

In the following, the MRAC-concept will be introduced and the effect of unmodeled dynamics on the stability proof is discussed in section III-D. A two-mass system is used as case study because it is the basic element of almost every mechatronic system. The mathematics for an arbitrary (linear, unknown) system are entirely analogous.

A. Model of the Plant

In Fig. 1, a schematic representation of a two-mass flexible servo system is given where $n_1(t)$ denotes the angular velocity of the motor and $n_2(t)$ the velocity of the load. The moments of inertia J_1 and J_2 as well as the viscous damping coefficient d and the stiffness c of the shaft are unknown. In most speed control systems, the motor torque is controlled by an inner control loop with negligible time constant such that a voltage $u(t)$ almost immediately leads to a torque $m_1(t)$ at the shaft of the two–mass system. The electrical components of the plant consisting of a power converter, a synchronous drive and a speed-sensor are therefore part of the parasitic dynamics which are not included into the model. Furthermore, all involved electrical parts have their own processor with different clock rates, i.e. there exists a time varying dead-time in the closed loop also not included into the model but problematic for the adaptive concept.

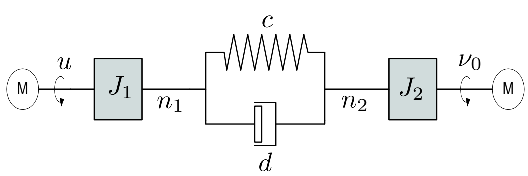

Fig. 1. The two-mass system

A continuous–time state space model of the mechanical part of the system representing the dominant dynamics is given by [5]:

$$\begin{bmatrix} \dot{n}_1 \\ \Delta\dot{\varphi} \\ \dot{n}_2 \end{bmatrix} = \begin{bmatrix} -\frac{d}{J_1} & -\frac{c}{J_1} & \frac{d}{J_1} \\ 1 & 0 & -1 \\ \frac{d}{J_2} & \frac{c}{J_2} & -\frac{d}{J_2} \end{bmatrix} \begin{bmatrix} n_1 \\ \Delta\varphi \\ n_2 \end{bmatrix} + \begin{bmatrix} \frac{u(t)}{J_1} \\ 0 \\ -\frac{\nu_0}{J_2} \end{bmatrix}$$

This is a standard model for a flexible coupling of two rotational masses, where $\Delta\varphi$ denotes the angle between the masses. In order to obtain a discrete-time model we substitute the 3-dimensional state vector of the discretized state space representation by 3 subsequent output measurements. After some standard calculation we obtain the following auto-regressive moving-average (ARMA) model of the two-mass system:

$$n_2(k+1) = \sum_{i=0}^{2} a_i n_2(k-i) + b_i u(k-i) + \xi(k+1) \quad (1)$$

Clearly, a_i, b_i, c_i are nonlinear functions of the physical parameters J_1, J_2, c, d of the plant. $\xi(k+1)$, in turn, accounts for all perturbations that have not been included into the model, i.e. the effect of the parasitic electrical part of the plant. For notational convenience we define

$$\theta_0 = [\ a_0 \ \ a_1 \ \ a_2 \ \ b_0 \ \ b_1 \ \ b_2 \ ]^T \quad (2)$$

$$\phi(k) = [\ n_2(k) \ \ n_2(k-1) \ \ n_2(k-2) \ \ u(k) \ \ u(k-1) \ \ u(k-2)]^T$$

and obtain

$$n_2(k+1) = \phi(k)^T\theta_0 + \xi(k+1) \quad (3)$$

B. Control Objective

Summing up the set of prior information about the discrete-time plant, we know that the system is

- of third order
- with delay $d = 1$
- minimum-phase

Given a reference model $H^*(z)$ with delay $d^* \geq 1$ and an arbitrary bounded input, e.g. $r(k) = n_{20} + sin(\omega\, k)$, the objective is to design a controller which tracks $n_2^* = H^*(z)\, r$ and keeps all signals in the system bounded.

In formal terms,

$$\lim_{k\to\infty} |n_2(k) - n_2^*(k)| \leq \varepsilon$$

$$\|\phi(k)\| < \infty \quad \text{for all } k > 0$$

where $\varepsilon > 0$ is some small value which is zero when $\xi = 0$. The second expression refers to boundedness of the regression vector ϕ which contains all signals in the system.

C. Design of the Adaptive Controller

Among the possible reference models we choose the simplest one, namely

$$H(z^{-1}) = z^{-1}$$

which results in $n_2^*(k+1) = r(k)$. With such a reference model we aim to design a deadbeat-controller which guarantees that the load speed n_2 is equal to its desired value only one instant of time later. The design of the adaptive control law proceeds as follows:

If the parameters were known we would set

$$n_2^*(k+1) = \phi(k)^T\theta_0 \quad (4)$$

and solve for $u(k)$. Since the parameters are unknown an identification model has to be built which generates estimates of θ_0. In view of equation (4) an obvious choice for such a model is

$$\hat{n}_2(k+1) = \phi(k)^T \hat{\theta}(k) \tag{5}$$

where $\hat{\theta}(\cdot) : \mathbb{Z}_0^+ \to \mathbb{R}^7$ represents a time–varying vector of parameter estimates:

$$\hat{\theta}(k) = [\ \hat{a}_0(k) \ \ \hat{a}_1(k) \ \ \hat{a}_2(k) \ \ \hat{b}_0(k) \ \ \hat{b}_1(k) \ \ \hat{b}_2(k)\ ]^T \tag{6}$$

$\hat{\theta}(k)$ is calculated from the system-model (3) under the assumption that unmodeled dynamics are not excited ($\xi(k) = 0$):

$$n_2(k) = \phi(k-1)^T \hat{\theta}(k) \tag{7}$$

If the assumption were true $\hat{\theta}(k)$ would lead to the same output $n_2(k)$ as θ_0. A standard recursive least squares algorithm is used to adjust the parameters which is both fast and robust to noisy measurements.

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{P(k-2)\phi(k-1)e_i(k)}{1 + \phi^T(k-1)P(k-2)\phi(k-1)} \tag{8}$$

where

$$P(k-1) = P(k-2) - \frac{P(k-2)\phi(k-1)\phi^T(k-1)P(k-2)}{1 + \phi^T(k-1)P(k-2)\phi(k-1)}$$

The difference between the identification model (5) and the model of the plant (3) leads to the identification error

$$e_i(k+1) = n_2(k+1) - \hat{n}_2(k+1) = \phi(k)^T \tilde{\theta}(k) + \xi(k+1) \tag{9}$$

where $\tilde{\theta}(k) = \theta_0 - \hat{\theta}(k)$ and contains the effects of both the parametric error $\tilde{\theta}$ as well as the residual error ξ due to unmodeled dynamics. As above, we set

$$n_2^*(k+1) = \phi(k)^T \hat{\theta}(k) \tag{10}$$

and solve for $u(k)$ to obtain the control law of the so called inverse controller. It is assumed that $\hat{\theta}(k)$ describes the system at time $k+1$ approximately as well as it did one step before when it was calculated ($n_2(k) = \phi(k-1)^T \hat{\theta}(k) = \phi(k-1)^T \theta_0$). If not – because the regression vector ϕ changed – new information about the parameters will be collected through the identification such that the described will be valid. The fact, that $\hat{\theta}(k)$ leads to the same output as the real vector θ_0 is referred to as the Certainty Equivalence Principle in adaptive control. It has the effect that the control error approaches the identification error asymptotically. In our case (since the plant has delay $d = 1$) the control error becomes

$$\begin{aligned} e(k+1) &= n_2(k+1) - n_2^*(k+1) \\ &= n_2(k+1) - \hat{n}_2(k+1) + \hat{n}_2(k+1) - n_2^*(k+1) \\ &= \phi(k)^T \tilde{\theta}(k) + \xi(k+1) = e_i(k+1) \end{aligned} \tag{11}$$

which is actually equivalent to the identification error (9). Hence, the controller inherits its stability properties from those of the identification procedure. Now if it can be shown that the identification error tends to zero ($e_i \to 0$), the control error will vanish too ($e \to 0$) and the control objective will be reached. But as already mentioned, the identification process is disturbed so it can be expected that the identification error will not vanish. In the following, this question should be discussed. In addition, it must be guaranteed that all signals of the nonlinear control loop (ϕ) keep bounded, i.e. the system is stable.

D. Discussion: Stability in Presence of Parasitic Dynamics

The stability of the MRAC-controller is linked to the dynamics of the estimation algorithm (8), since the identification error equals the control error at every instant of time, i.e. $e_i(k) = e(k)$ for all $k > 0$. According to the standard proof for MRAC [4] the Lyapunov-function

$$V(k) = \tilde{\theta}(k)^T P(k-1)^{-1} \tilde{\theta}(k)$$

is considered. To guarantee stability of the estimation algorithm, the quantity $\Delta V(k) = V(k) - V(k-1) \leq 0$ must be non-positive. In [7] the Lyapunov-function $V(k)$ respectively $\Delta V(k)$ was calculated for the case of unmodeled dynamics, i.e. $\xi \neq 0$:

$$\Delta V(k) = \frac{-e(k)^2}{1 + \phi(k-1)^T P(k-2)\phi(k-1)} + \xi(k)^2 \tag{12}$$

At this point, it is obvious that excitation of parasitic dynamics cause $\xi^2(k)$ to increase and $\Delta V(k)$ to become positive. Hence $V(k)$ may increase and the identification process becomes unstable. Experimental studies demonstrate that even an unconsidered dead-time of half a sample period leads to an unstable system (Fig. 3, left side) – consequently, model reference adaptive control is just theory and is not directly applicable to real systems.

An extension of MRAC is needed such that an excitation of unmodeled dynamics is suppressed after a finite time. The reason for this initial excitation – as mentioned in section II – can be found in the nonlinear identification algorithm initialized with a wrong parameter-vector $\hat{\theta}(0)$ that ends up in a vicious circle as described above. If the effect of unmodeled dynamics were to vanish after a finite time because of an appropriate extension of the MRAC-concept we could approximate $\xi^2 \approx 0$ for $k > k_1$ and some constant $k_1 > 0$. The effect is large during a transient phase at the beginning of control action. In the worst case we obtain,

$$\Delta V(k) = \begin{cases} > 0 & \text{initially, i.e. for } 0 \leq k \leq k_1 \\ \leq 0 & \text{otherwise, } \ k > k_1. \end{cases}$$

Hence, $\Delta V(k)$ is negative semidefinite for all k except a finite number. In [7], it is shown that under that assumption the standard stability proof [4] holds. Hence, if the system is minimum-phase, $\phi(k)$ does not grow without bound and $e_i(k) \to 0$ as $k \to \infty$.

Consequently, if there exists an extension of the MRAC-concept that prevents excitation of unmodeled dynamics after a finite time the adaptive controller is stable and guarantees a control error $e(k) = e_i(k) \to 0$ for $k \to \infty$, i.e. perfect tracking even in real applications. In the following, a gain-controller is presented as a possible way to achieve this.

IV. Robust Model Reference Adaptive Control: Gain-Control Extension

A. Idea of the Gain-Controller within MRAC

In section II, the problem was stated. In a linear setting, unmodeled dynamics causes much less trouble since the excitation of the system is directly linked to the frequency-content of the desired value $n_2^*(k)$. Consequently, the frequency of the control output $u(k)$ is bounded above and hence it is known what the order of a model should be in order to represent the dominant dynamics of the system.

With use of the gain-controller, this consideration is made applicable to adaptive control where in contrast the nonlinear nature of the identification process leads to an arbitrarily high frequency content of the control output $u(k)$ independent of the frequency content of the desired value $n_2^*(k)$. If the activity of the actual value $n_2(k)$ is greater than the one of the desired value $n_2^*(k)$ it can be expected that the frequency content of $u(k)$ is too high, meaning that unmodeled dynamics lead to undesired control-behavior exciting itself. In this case, the amplitude of the high frequency control output has to be reduced such that parasitic dynamics is not excited anymore. For that, the high-frequency gain is continuously reduced by the factor $0 < g(k) \leq 1$ regulated by the gain-controller. Consequently there exists a time k_1 when excitation of unmodeled dynamics is cut off: $\xi(k_1) = 0$. This is the requirement for the stability analysis discussed in section III-D. Afterwards the identification process works undisturbed and leads to useful estimates of $\hat{\theta}$. Now, a better control output is calculated that perhaps ends in a too slow behavior of the actual output when compared to the desired one because of a too small high-frequency gain. The amplitude of $u(k)$ should be increased by raising the factor $g(k)$ until the control objective is reached.

If the order of the model represents the dominant dynamics of the system, referred to the desired value $n_2^*(k)$, parasitic dynamics is not be excited any more when tracking the reference signal. In this case the high-frequency gain is increased up to its old value, i.e $g(k) = 1$. That means, the gain-controller is no longer active. This arises from the fact that the estimation process has converged such that a linear behavior of the control loop appears and no unexpected frequencies show up in the control output as in the transient phase of identification.

If the order/relative degree of the model do not represent the dominant dynamics of the system because of unconsidered dead-times, where a slowly signal-change already leads to errors, excitation of unmodeled dynamics is at first just minimizable but can be compensated as well with the gain-controller concept. In this case, the high-frequency gain is increased up to a value smaller than its old one, i.e. $g(k) < 1$, such that the remaining $\xi(k)$ will be compensated by reducing the amplitude of the control output. Now in detail. Dead-time causes the main problem because this parasitic dynamics is always visible in the case of signal changes - if the gradient of the signal is high, $\xi(k)$ is large. In the case of a slow change of the signal, $\xi(k)$ is small but nonzero. ξ varies with the gradient of signals. Imagine, if the actual value is measured later than expected through the model the inverse controller calculates a control output that is too large – the actual value is nearer to the desired value as expected through the measurement. Consequently the control output $u(k)$ is greater than needed when the control error $e(k) = 0$ is already zero at time k . This results in an oscillating control output $u(k)$ that means further excitation of unmodeled dynamics. Now, if the factor $g(k)$ remains smaller than one ($g(k) \neq 1$) an excessive $u(k)$ will be reduced such that the effect of dead-times, even slowly changing dead-times, are compensated adaptively in order to ensure that $\xi(k) = 0$ for all $k > k_1$. This is the time k_1 when no parasitic dynamics is excited anymore.

B. Realization of the Gain-Controller Concept

The gain-controller is used to monitor and adapt the loop-gain of the MRAC-controller. For this end, the gain-controller is placed between the control output of the inverse controller and the input of the plant as shown in Fig. 2. In the following, the nonlinear control law is presented:

$$v(k) = g(k)\, u(k) \tag{13}$$

in which the factor

$$g(k) = g(k-1) + [a(k) \cdot |e(k)| - b(k)] \tag{14}$$

shows integral behavior with $a(k)$ and $b(k)$ affecting the slope of the integrator:

$$a(k) = a \quad \wedge \quad b(k) = 0 \tag{15}$$

$$\text{for} \quad \begin{cases} n_2^*(k) - n_2^*(k-1) > 0 & \wedge \quad e(k) > 0 \\ n_2^*(k) - n_2^*(k-1) < 0 & \wedge \quad e(k) < 0 \\ n_2^*(k) - n_2^*(k-1) = 0 & \wedge \quad e(k) \cdot n_2^*(k) > 0 \end{cases} \tag{16}$$

$$a(k) = 0 \quad \wedge \quad b(k) = b \tag{17}$$

$$\text{for} \quad \begin{cases} n_2^*(k) - n_2^*(k-1) > 0 & \wedge \quad e(k) < 0 \\ n_2^*(k) - n_2^*(k-1) < 0 & \wedge \quad e(k) > 0 \\ n_2^*(k) - n_2^*(k-1) = 0 & \wedge \quad e(k) \cdot n_2^*(k) < 0 \end{cases} \tag{18}$$

$$a(k) = 0 \quad \wedge \quad b(k) = 0 \tag{19}$$

$$\text{for} \quad \{ \; e(k) \approx 0 \tag{20}$$

The constants a and b have the property

$$b \gg a > 0 \tag{21}$$

and concerning $g(k)$ the auxiliary condition

$$0 < g(k) \leq 1 \qquad \forall\, k > 0 \tag{22}$$

must hold.

The gain-controller works as follows: at every instance of time the desired and actual value are compared. Out of this control error $e(k) = n_2^*(k) - n_2(k)$ it will be obvious if the actual evolution of $n_2(k)$ lags or leads the evolution of the desired value $n_2^*(k)$.

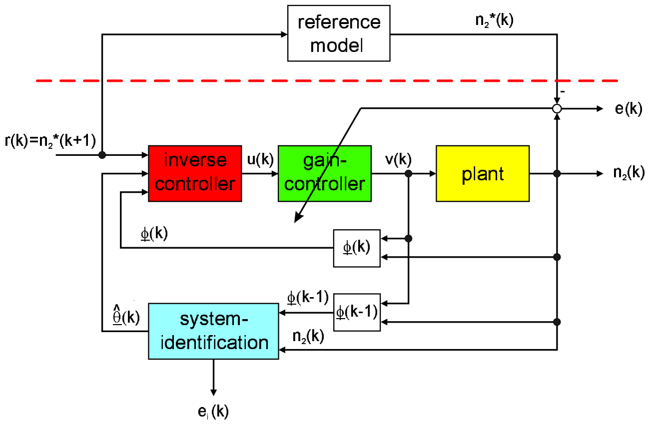

Fig. 2. Scheme: Gain-controller within MRAC

If $n_2(k)$ leads – as expressed in (18) – the activity of the actual value $n_2(k)$ is greater than the one of the desired value $n_2^*(k)$. According to the proposed idea in section IV-A the loop-gain should be decreased to avoid excitation of unmodeled dynamics, i.e. the factor $g(k)$ will be decreased such that a value smaller than one will be reached: $g(k) < 1$. To this end, (14) is calculated according to (17).

If $n_2(k)$ lags – see (16) – the loop-gain is too small. Consequently, the factor $g(k)$ should be increased such that the control objective will be fulfilled. To this end, (14) is calculated according to (15).

If the actual value $n_2(k)$ oscillates around the desired value $n_2^*(k)$ the actual value leads and lags periodically. This would result in a constant mean value of the factor $g(k)$ if the increase and decrease mechanism operates with the same gradient. That means that the high-frequency gain will not be decreased even if the oscillation may be traced back to excitation of parasitic dynamics. Because of this, it is important to make the decrease faster than the increase. To this end, condition (21) must hold. The gradient of the increase of the factor $g(k)$ is defined by the constant a, the gradient of the decrease by the constant b, i.e these constants determine the sensitivity of the gain-controller.

Holding condition (21) is advisable to guarantee robustness of the gain-controller itself. If excitation of unmodeled dynamics is present the time k_1 (i.e. where no parasitic dynamics is excited) should be reached as fast as possible to break the vicious circle within a short transient phase, i.e. b should be large in magnitude. The following fine tuning because of better parameter estimates is a slow process compared to the stabilization process. In addition, gain-increase is generally more critical than gain-decrease (for controlling stable systems). Imagine that the quantization of the discrete increase process is not fine enough because of a too great gradient the high frequency-gain may quickly become too large causing again an excitation of unmodeled dynamics. Hence, summing up all the requirements, a should be of small value and weighted with the control error $e(k)$ – see (14) – to guarantee an approach to the ideal value of $g(k)$ with high resolution.

As soon as the desired value is reached and $e(k) \approx 0$ – as expressed in (20) – the gain-adaption will be turned off. According to (14) combined with (19) the high-frequency gain has converged to a new value guarantying applicability of model reference adaptive control. Now, perfect performance of tracking is just the result of MRAC-theory.

C. Discussion: Stability of the Overall System

According to the stability analysis in section III-D an extension of MRAC was developed that guarantees the existence of a time k_1 where no unmodeled dynamics is excited anymore as discussed in section IV-A. The main idea is to reduce the amplitude of the control output what never results in instability if the system itself is stable. Because of the auxiliary condition (22) the control output will never be greater than calculated by the MRAC-controller without the extension. The factor $g(k)$ is adjustable between a value equal to one and greater than zero. Thus the maximum allowable value $g(k) = 1$ leads just to an unaffected MRAC-controller. Further, the estimation process and calculation of the MRAC control output $u(k)$ work independently of the activity of the gain controller (cp. section III-C and Fig. 2) such that the combination MRAC and gain-controller does not result in instability. The only important point is that the gain should always be greater than zero in order to hold the identification process alive. In the end, because of $b \gg a$ the amplitude of the control output will be reduced as soon as instability – detected by the phase lead or lag of the plant output – appears.

It is clear that reducing the amplitude of the controller may cause problems if the system is unstable – the manipulated variable may not be sufficient to stabilize the system if the gain is increased too slowly after a decrease. It depends on the dynamics of the gain-controller if an unstable system can be stabilized. But one may argue that model reference adaptive control is improper for controlling an unstable system. Although theoretically possible in industry this fact is absolutely uninteresting because the system may already be damaged in the transient phase. On the other hand, MRAC is very interesting for the control of stable damped systems like servo drives. In this case, a reduction of the gain leads generally to a stable system particulary if excited oscillations are damped. Then, the proposed extension of the MRAC-principle leads always to a stable control loop with smooth control output and perfect tracking with control error zero.

V. Simulation Results

The two-mass system presented in section III-A should be controlled with the MRAC-concept because all parameters are unknown. The motor activates the load over a shaft with the aim to control the speed $n_2(k)$ of the load – here appearing oscillations caused by the stiffness of the shaft should be damped. For the simulation a two-mass system with speed sensor is considered whereas the speed sensor is neglected as parasitic dynamics. Usually, the speed is

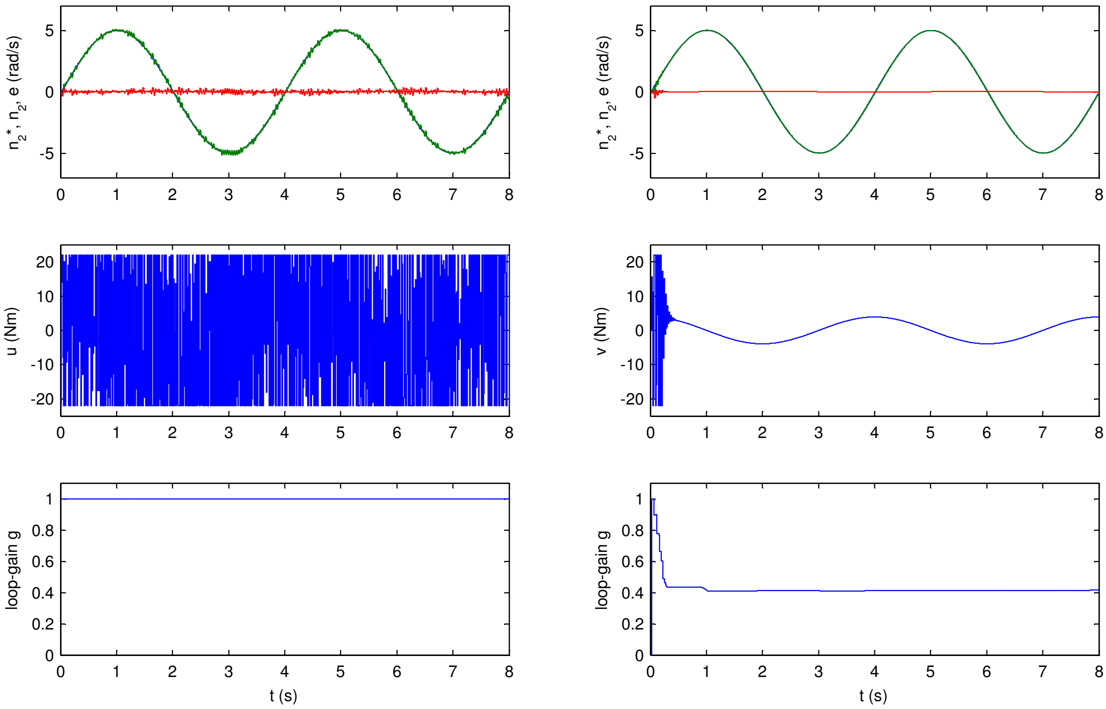

Fig. 3. Adaptive control of a two-mass system in the presence of unmodeled dynamics; left: unrobust/unstable behavior of the MRAC-concept, right: robust/stable behavior of MRAC extended by the gain-controller

calculated by the measured position φ (encoder):

$$n_2(k) = \frac{\varphi(k) - \varphi(k-1)}{h} \tag{23}$$

with sampling period h. Effectively this equals a dead-time of $\frac{h}{2}$. In spite of that relatively small unmodeled dynamics referred to the slowly alternating sine as reference signal $n_2^*(k)$ the control output $u(k)$ respectively actual value $n_2(k)$ shows intolerable aggressive behavior exciting parasitic dynamics (Fig. 3, left side) – without a saturation of the control output instability would appear.

If the MRAC-concept is extended by the gain-controller (Fig. 3, right side) excitation of unmodeled dynamics is cut off as described above in detail. At the beginning of the simulation, like before, a high frequency control output shows up because of the nonlinear estimation process and the neglected dead time. Immediately, the high frequency gain respectively the factor $g(k)$ will be decreased, such that after a short transient phase, the control output $v(k)$ as well as the actual value $n_2(k)$ show a smooth behavior. Even the high frequency gain keeps reduced to compensate for the effect of the unconsidered dead time perfect tracking according to the desired dead-beat behavior occurs – the control error vanishes: $e(k) = 0$.

VI. Conclusion

In the present paper, the robustness of model reference adaptive control (MRAC) to parasitic dynamics is investigated experimentally. It is seen, that even a dead time of half a sampling period must not be neglected. Hence MRAC is not applicable in real scenarios where the sensors alone typically have such dead times. Due to the nonlinearity of the adaptive system the effect of unmodeled dynamics on the stability cannot be quantified easily. In this paper, a pragmatic viewpoint has been taken, which in turn, leads to excellent results in the experimental study. A modification of the standard adaptive scheme is proposed which aims at re–covering the assumptions made in the standard proof of stability (i.e. that no unmodeled dynamics are present). The idea is to detect whether unmodeled dynamics are excited or not through the phase lead or lag of the output signal. If the phase leads the gain $g(k)$ is reduced which also results in a reduction of the excitation. While this cannot be justified quantitatively, notice that in the limit $g(k) \to 0$ no excitation occurs at all. In other words, while it is hard to quantify the error due to unmodeled dynamics, their effect can be made arbitrarily small.

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