A Controller for Changing the Yaw Direction of an Underactuated Unicycle Robot

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Abstract - Several researchers have developed unicycle robots and associated control systems. We have also developed a unicycle robot that consists of a body, a disk driven by a dc motor for rotation, and a wheel driven by another dc motor to move the body. Because the disk is rotated in the frontal plane, there is no direct force to change the yaw direction of the robot. Hence conventional controllers cannot change the yaw direction. The present paper proposes a control method for changing the yaw direction by using the interaction between individual motions in the yaw, pitch, and roll directions. Simulation and experimental results show that the control method can control the yaw direction of the unicycle robot while stabilizing its posture. An observer to estimate the robot posture is also developed.

Index Terms – Unicycle Robot, Underactuated System, Regulator, Nonlinear Control System

I. INTRODUCTION

Various unicycle robots have been developed in a number of studies, and several control systems have been proposed for these robots [1],[2],[3]. In 1984, a One Wheel-Locomotive Robot was developed by Honma *et al.*[4]. The postural stability of the robot was achieved by gyro precession. Furthermore, Z. Sheng and K. Yamafuji developed a unicycle robot that was driven using four motors. Its postural stability control was realized through a turntable rotating in the horizontal plane [5]. A toy "Loony Cycle" was also developed as a radio controlled unicycle robot. Two lateral-thrusting fans applied a sideways force to the axle that made the wheel lean, making the robot turn automatically [6].

We have also developed a unicycle robot, which consists of a body, a disk driven by a dc-motor, and a wheel driven by another dc-motor to move the body. The robot has only two motors. The disk is rotated in the frontal plane, and there is no direct force to change the yaw direction of the robot. Thus, changing the yaw direction of the unicycle robot while stabilizing its posture is difficult. Based on the experience of riding a unicycle, it is prospected that slanting the body at a desired angle can control the yaw direction of the moving unicycle robot. However, controlling the developed unicycle robot by tilting its posture at a constant angle is difficult. The tilted robot easily falls over. Thus, a different method must be developed. In the present paper, we propose a control method for changing the yaw direction by using the interaction between individual motions in the yaw, pitch, and roll directions.

This paper is organized as follows. In Section 2, equations representing the motion of the unicycle robot are derived. In Section 3, a control method that uses state feedback for stabilizing the posture and sinusoidal inputs for exerting the interaction forces to change the yaw angle are proposed. The parameters of the controller are obtained using simulation results with the derived equations. Section 4 describes the principle of an observer developed to estimate the robot posture. The control method is confirmed experimentally in Section 5, and conclusions are presented in Section 6.

II. MODEL

A. *Unicycle robot*

Fig. 1 shows a model of a unicycle robot that consists of a body, a disk rotated in the rolling (lateral) direction by a dc-motor, and a wheel rotated in the pitch (longitudinal) direction by another dc-motor. Fig. 1 also shows the coordinate system used to describe the posture of the robot. The coordinate (x,y) is the point at which the wheel contacts the floor, and the posture of the robot is indicated by the roll angle β, the pitch angle ϕ, and the yaw angle α.

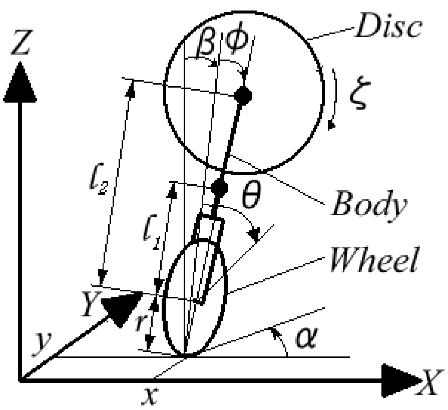

Fig. 1 Model of the unicycle robot

Other variables and parameters shown in Fig. 1 are as follows: θ is the wheel angle, ς is the disk angle, r is the radius of the wheel, l_1 is the distance between the center of gravity of the body and the axis of rotation of the wheel, and l_2 is the distance between the axis of rotation of the disk and the axis of rotation of the wheel.

B. Equation of Motion

Equations representing the motion of the unicycle robot are derived using the Lagrange equations of motion, with state variables $x_1 = \alpha$, $x_2 = \dot{\alpha}$, $x_3 = \phi$, $x_4 = \dot{\phi}$, $x_5 = \theta$, $x_6 = \dot{\theta}$, $x_7 = \beta$, $x_8 = \dot{\beta}$, $x_9 = \varsigma$, and $x_{10} = \dot{\varsigma}$. The parameters m_w, m_b, and m_d denote a mass of the object, d_w and d_d denote a friction coefficient, and n_w and n_d denote a gear ratio of motor, where the subscript w means the wheel, the subscript b means the body, and the subscript d means the disk, respectively. The parameter d_a is a friction coefficient between the wheel and the floor. The parameters I_{bx}, I_{by}, and I_{bz} denote a moment of inertia of the body about X-axis, about Y-axis, and about Z-axis, respectively.

The derived equations are very complex and such complex equations have been frequently simplified by linearization. However, the linearized equations are independent for each postural angle. Therefore, in the present paper, we simplified the equations by neglecting higher order or second order terms of the velocities under the assumption that the velocities are small. Sinusoidal functions in the equations were not linearized. As a result, the simplified equations cannot express centrifugal or Coriolis forces, however, can represent the interactions between individual motions in the yaw, pitch and roll directions. Hence, we used this simplified model to simulate the robot motion in order to determine the control method and confirm its performance.

The state equation for x_1 and x_2 are as follows:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \{(-rN_1(gA_3N_1 \cos x_7 \sin x_3 + d_a(A_4 + rN_1)x_2) \\
&\quad + A_2(A_3(D + gN_1 \cos x_7 \sin x_3 - n_p u_p) - \\
&\quad rN_1 \sin x_7 (D + gN_1 \cos x_7 \sin x_3 - n_p u_p) \\
&\quad + A_4(-D \sin x_7 + n_p u_p \sin x_7 + d_a x_2))\} / \\
&\quad \{(A_2^2 A_4 \sin^2 x_7 \\
&\quad + A_2(A_3^2 - A_1A_4 - 2rN_1A_3 \sin x_7) \\
&\quad + rN_1(-A_3^2 + A_1(A_4 + rN_1))\}
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
A_1 &= I_{dy} \sin^2(x_7 + x_9) + I_{dz} \cos^2(x_7 + x_9) \\
&\quad + (I_{bz} + I_{wx}) \cos^2 x_7 \\
&\quad + \sin^2 x_7 (I_{by} + I_{wy} + r^2 M + 2rN_1 + N_2) \\
A_2 &= I_{wy} + r^2 M + rN_1 \\
A_3 &= I_{dy} \cos \varsigma \sin(x_7 + x_9) - I_{dz} \cos(x_7 + x_9) \sin x_9 \\
&\quad + (I_{by} + rN_1 + N_2) \sin x_7 \\
A_4 &= I_{dy} \cos^2 x_9 + I_{dz} \sin^2 x_9 + I_{by} + N_2 \\
A_5 &= I_{dx} + I_{bx} + I_{wx} + r^2 M + 2rN_1 + N_2 \\
D &= d_w(x_6 - x_4) \\
M &= m_w + m_b + m_d \\
N_1 &= m_b l_1 + m_d l_2 \\
N_2 &= m_b l_1^2 + m_d l_2^2
\end{aligned}
$$

Equations for the other state variables are similar to those given above.

III. Proposed Control Method

A. State Feedback and Sinusoidal Wave Input

In this section, we propose a control method for changing the yaw direction of the unicycle robot while stabilizing its posture using only the two motors. State feedback control is required in order to stabilize the posture. In order to change the yaw direction, the interactions between individual yaw, pitch and roll motions are used. In order to strengthen the interactions, sinusoidal inputs are adopted and highly-effective parameters of the sinusoidal inputs are obtained by simulating the robot motion using the equations derived in Section II and the identified parameters of the developed robot, as shown in Table I.

The input u_p for the motor that drives the wheel, i.e., for the pitch direction, and the input u_r for the motor that drives the disk, i.e., for the roll direction, are as follows:

$$
\begin{aligned}
u_p &= k_3x_3 + k_4x_4 + k_6x_6 - h_p \sin(\omega t + T) \\
u_r &= k_7x_7 + k_8x_8 + k_{10}x_{10} - h_r \sin(\omega t)
\end{aligned}
\tag{2}
$$

In order to determine the value of the parameters of sinusoidal elements in (2), we examined the performance of the control of the yaw direction by changing the following parameters: h_p, h_r, the magnitudes of the sinusoidal elements; ω, the angular velocity; and T, the phase shift between two sinusoidal elements. First, the slope of the time response of the yaw angle was found to be approximately constant. We then chose the slope, i.e. the rate of yaw, as the index of the performance.

TABLE I
PARAMETERS OF THE MODEL

$m_w = 0.06[kg]$	$m_b = 0.848[kg]$
$I_{wx} = 1.45\times10^{-5}[kgm^2]$	$I_{bx} = 1.52\times10^{-2}[kgm^2]$
$I_{wy} = 2.89\times10^{-5}[kgm^2]$	$I_{by} = 1.52\times10^{-2}[kgm^2]$
$I_{wz} = 1.45\times10^{-5}[kgm^2]$	$I_{bz} = 1.77\times10^{-2}[kgm^2]$
$l_2 = 0.177[m]$	$l_1 = 0.10[m]$
$n_w = 19.0$	$n_d = 19.0$
$m_d = 0.384[kg]$	$r = 0.031[m]$
$I_{dx} = 4.30\times10^{-4}[kgm^2]$	$d_a = 6.0\times10^{-2}[Nm/(rad/s)]$
$I_{dy} = 2.15\times10^{-4}[kgm^2]$	$d_w = 2.0\times10^{-4}[Nm/(rad/s)]$
$I_{dz} = 2.15\times10^{-4}[kgm^2]$	$d_d = 1.0\times10^{-4}[Nm/(rad/s)]$

The following relationship between the parameters in the sinusoidal elements and the rate of yaw are obtained:

1. Effect of ω

   The rate of the yaw angle depends on the angular velocity ω, and the maximum rate is obtained when ω = 0.67 [Hz]. Fig. 2 shows the rate of yaw with respect to ω when h_p = 1.0 [Nm], h_r = 1.5 [Nm], and T = 0.

2. Effect of T

   Fig. 3 shows the rate of yaw with respect to T when h_p = 1.0 [Nm], h_r = 1.5 [Nm], and ω = 0.6 [Hz]. From this figure, the maximum negative rate is obtained when the phase T = 0 [rad] and the maximum positive rate is obtained when $T = \pi$, and the yaw angle does not change when T = 1.5 or T = 4.65 [rad]. This result suggests that we can control the yaw direction by modifying the phase T.

3. Effect of Amplitude h_p and h_r

   The simulations were carried out maintaining parameters ω and T constant, i.e., ω = 0.6 [Hz] and T = 0 [rad] or ω = 0.6 [Hz] and T = π [rad], in order to examine the effects of amplitude h_p or h_r under the three conditions described below:

   1) Change h_p from 0 to 15 [Nm], while maintaining h_r = 1.5 [Nm].
   2) Change h_r from 0 to 15 [Nm], while maintaining h_p = 1.0 [Nm].
   3) Set $h_p = h_r$ and change them from 0 to 15 [Nm].

   In all cases, as the amplitude of the sinusoidal input, h_p or h_r, increases, the change in the yaw angle increases under the assumption that the amplitude is less than 15 [Nm]. For example, Fig. 4 indicates the rate of the yaw with respect to h_p under the third condition and T = 0 [rad].

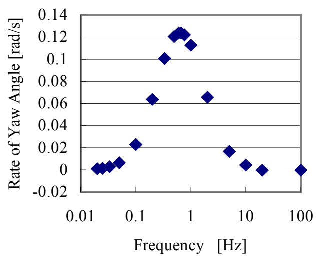

Fig. 2 Effect of angular velocity of sinusoidal elements on rate of yaw

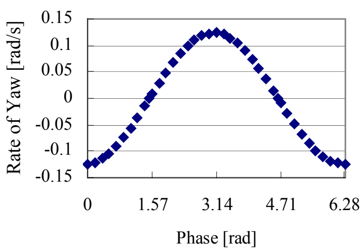

Fig. 3 Effect of phase difference T on rate of yaw

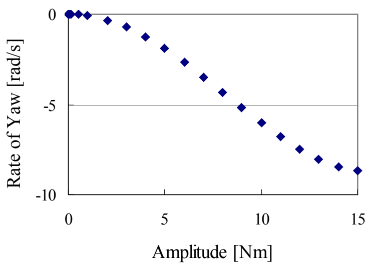

Fig. 4 Effect of the amplitude of sinusoidal elements on rate of yaw

Based on these simulation results, the parameters in the sinusoidal elements are determined as follows:

$$
\begin{aligned}
&h_p = \sqrt{k_{hp}|x_1|} \quad [Nm] \quad (0 \le h_p \le 1.0) \\
&h_r = \sqrt{k_{hr}|x_1|} \quad [Nm] \quad (0 \le h_r \le 1.5) \qquad (3) \\
&T = T_0 - k_T x_1 \quad [rad] \quad (0 \le T \le \pi) \\
&\omega = 0.67 \quad [Hz]
\end{aligned}
$$

The phase T_0 is set to 1.51 [rad] in order to maintain the change in the yaw angle as zero when x_1 = 0. The amplitude h_p or h_r changes with the magnitude of x_1 in order to improve the convergence of state variables, and the amplitudes are limited in order to prevent the robot from falling over. Parameters k_{hp} and k_{hr} are constant.

Fig. 5 shows the time response of the yaw angle of the robot controlled with the operating inputs described in (2) and (3). The yaw angle reaches the desired value x_1 = 0 [rad] from the initial value $x_1(0) = \pi/2$, while the pitch angle and the roll angle also reach the desired value of 0, as shown in Fig. 6. The simulation results show that the proposed control method can change the yaw direction of the unicycle robot to the desired value while stabilizing its posture.

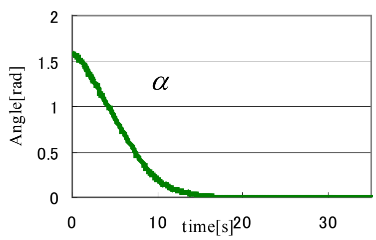

Fig. 5 Time response of the yaw angle in controlled robot on simulation

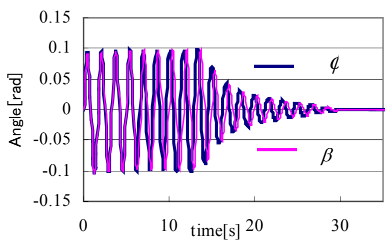

Fig. 6 Time responses of the pitch and roll angles in controlled robot

IV. OBSERVER

A. Experimental Equipment

The developed unicycle robot is shown in Fig. 7. The robot consists of a body, a disk driven by a dc-motor, and a wheel driven by another dc-motor. The control signals are transferred from a PC to two motors via wires. In order to estimate the posture angle, a gyro sensor is attached to the robot for each yaw, pitch and roll angle. Each angle is estimated by integrating the measured value and the estimated values have drift terms. In order to compensate for the drift terms, an observer is developed with two accelerometers attached to the robot.

B. Observer to estimate posture angles

The values measured using the gyro sensor have high accuracy in the high frequency range but low accuracy in the lower frequency range. On the other hand, the values measured using the accelerometer involve the acceleration due to the robot motion. Therefore, the accelerometer has high accuracy in the low frequency range but low accuracy in the high frequency range, because the robot vibrates at high frequency.

First, we explain the reason for using two accelerometers to estimate the postural angles. Fig. 8 shows a pendulum with two accelerometers and a potentiometer. The potentiometer is attached to a fixed point in order to measure the angle θ directly for comparison to the angle estimated from the accelerometer measurement. The measurement value of Accelerometer 1 attached at a point B, at a distance of L [m], is set to V_1 [v], and that of Accelerometer 2 attached at a point C, at a distance of μL [m], is set to V_2 [v]. When the acceleration measurement is affected by only a gravity force, the measurement V_1 and V_2 depend on only the posture angle θ, then they are represented as follows:

$$V_1 = a_1 \sin\theta + b_1, \quad V_2 = a_2 \sin\theta + b_2 \qquad (4)$$

Fig. 9 (a) shows the angle θ estimated from (4) when the head of the pendulum is released after bringing up to a point.

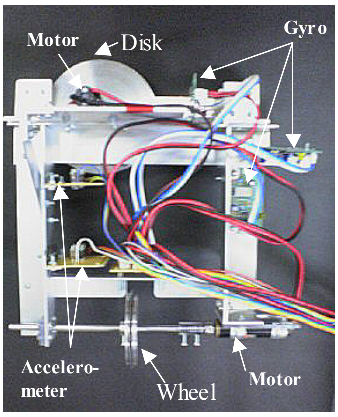

Fig. 7 Developed unicycle robot

They both represent a free damped harmonic oscillation, but they have smaller amplitudes than the value measured using the potentiometer because V_1 or V_2 depends on not only a gravity force but also the dynamics of the pendulum. The main factor of the dynamics is an angular acceleration. The angular acceleration at a point B is $L\ddot{\theta}$ and that at a point C is $\mu L\ddot{\theta}$, hence, the voltage due to the latter angular acceleration is μ times as great as the voltage A_1 due to the former angular acceleration. Adding these terms to (4), actual measurements V_1 and V_2 are obtained.

$$V_1 = A_1 + a_1 \sin\theta + b_1, \quad V_2 = \mu A_1 + a_2 \sin\theta + b_2 \quad (5)$$

From (5), we have

$$\theta = \sin^{-1}\left(\frac{(V_2 - b_2) - \mu(V_1 - b_1)}{(a_2 - \mu a_1)}\right). \qquad (6)$$

The parameters a_1, a_2, b_1 and b_2 are measured previously before the measurement of angle θ. A posture angle $\theta(t)$ at a time t is determined from (6) using $V_1(t)$ and $V_2(t)$ measured at each sampling time t. Fig. 9 (b) shows that the value estimated from (6) coincides with the value measured using the potentiometer. Thus the effects due to the dynamics of the pendulum can be cancelled by this method. But, when two accelerometers are attached to the unicycle robot for estimating the posture angle, the effects of more rapid dynamics due to the vibration motion of the robot remain. To reduce this effects, we develop an observer.

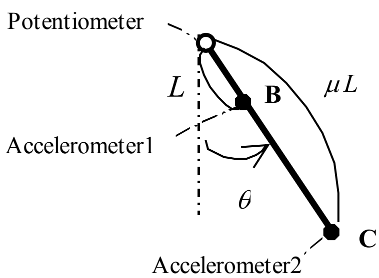

Fig. 8 Pendulum for confirming the method for estimating posture

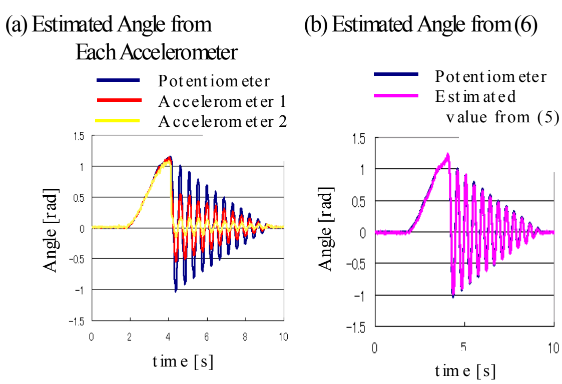

Fig. 9 Angle measurements in a free damped harmonic oscillation

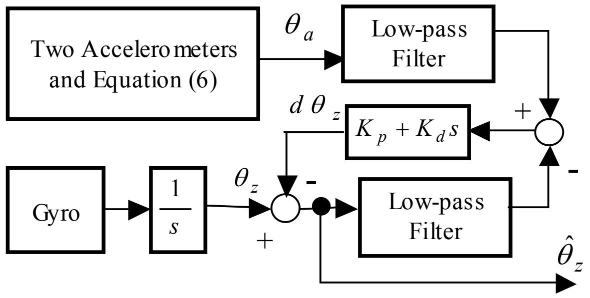

Fig. 10 Observer to estimate posture angle

Next, we develop an observer for estimating the postural angles θ of the unicycle robot by combing the value measured using a gyro sensor and the value estimated from (6), as shown in Fig. 10. The angle value θ_a estimated from (6) is passed through a low-pass filter and the angle value θ_z measured using the gyro sensor is also passed through a low-pass filter. The difference between the two values and the derivative of the difference are negatively fed back and are added to the measured value θ_z. This sum value is the estimated angle $\hat{\theta}_z$. The accelerometers used in this experiment can measure both the pitch angle and the roll angle. For estimating the pitch angle, θ_a in the observer is set to the value estimated from (6) using the pitch elements in the accelerometers and θ_z is set to the value measured using the gyro sensor for the pitch angle. Similarly, for estimating the roll angle, θ_a in the observer is set to the value estimated from (6) using the roll elements in the accelerometers and θ_z is set to the value measured using the gyro sensor for the roll angle. The yaw angle is determined using only gyro sensor measurement.

V. Experiment

A. Stabilizing Control

First, we will confirm that the posture of the unicycle robot can be stabilized by a state feedback controller using the developed observer. Fig. 11 shows the time responses of the pitch angle and the roll angle when the robot was pushed in the pitch direction 5 [s] after standing still. The disturbed posture returns to the ordinal still state. The result shows that the developed observer estimates the pith and roll angles correctly.

B. Control for Changing Yaw Direction

Although the control method proposed in (2) provided good performance in the simulation, in the experiment, the robot could only be controlled well when the robot was moving forward. When the robot ran backward, the wires prevented the robot from moving freely. Therefore, a force term for moving the robot forward was added to the input u_p, as follows:

$$u_p = k_3 x_3 + k_4 x_4 + k_6 (x_6 - v_p) - h_p \sin(\omega t + T) \qquad (6)$$

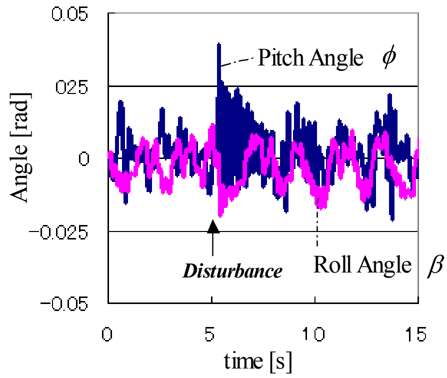

Fig. 11 Response when disturbance applied to the pitch direction

Using the modified input u_p shown in (6) and u_r in (2), the effects of phase T of sinusoidal input are examined, where the amplitudes of the sinusoidal inputs are set to be constant and h_p = 0.02 and h_r = 0.08. The angular velocity ω of the sinusoidal input is 0.67 [Hz], and the velocity v_p is 0.5 [rad/s]. In the experiments, the robot was standing with the assistance of the state feedback controller. After 5 [s], the sinusoidal inputs were applied to change the yaw direction. Fig. 12 shows the time response of the robot when $T = 0$, and Fig. 13 shows that when $T = \pi$. Similar to the simulation results, the yaw rate of α is negative when $T = 0$ and is positive when $T = \pi$.

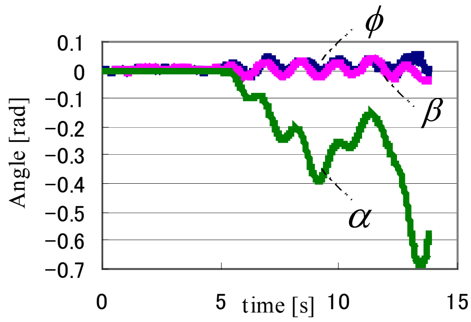

Fig. 12 Time response of three postural angles when $T=0$

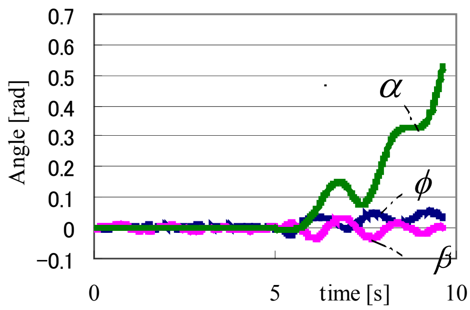

Fig. 13 Time response of three postural angles when $T=\pi$

Finally, the performance of the proposed control method was confirmed using the following inputs:

$$u_r = k_7 x_7 + k_8 x_8 + k_{10} x_{10} - h_r \sin(\omega t)$$
$$u_p = k_3 x3 + k_4 x_4 + k_6 (x_6 - v_p) - h_p \sin(\omega t + T) \quad (7)$$

where

$$h_p = \sqrt{k_{hp}|x_1|} \quad [Nm] \quad (0 \le h_p \le 0.02)$$
$$h_r = \sqrt{k_{hr}|x_1|} \quad [Nm] \quad (0 \le h_p \le 0.08)$$
$$T = 1.51 - k_T x_1 \quad [rad] \quad (0 \le T \le \pi)$$
$$\omega = 0.67 \quad [Hz]$$
$$v_p = 0.25 \quad [rad]$$

Fig. 14 shows the experimental result in which the initial value of the yaw angle was set to $x_1(0) = -\pi/6[rad]$ and the desired value of the yaw angle was set to $x_1 = 0[rad]$. First, the robot was standing with the assistance of the state feedback controller. After 5 [s], the sinusoidal inputs were applied to change the yaw direction. The yaw angle increased from $x_1(0) = -\pi/6[rad]$ beyond the desired value of zero and then tended to return to the desired value, while both the pitch and roll angles remained at approximately 0, the desired values. The time responses of the amplitude of the sinusoidal elements, h_p and h_r, and that of the phase, T, are also shown in Fig. 14. The amplitudes tend toward zero, and the phase tends toward 1.5 [rad], at which the yaw angle does not change.

This result demonstrates that the proposed control method can change the yaw direction of the unicycle robot while stabilizing its posture. However, the motion of the robot was often disturbed, because the weight of the wires was large compared to that of the body. In the future, we will improve the unicycle robot so that the control information can be sent wirelessly.

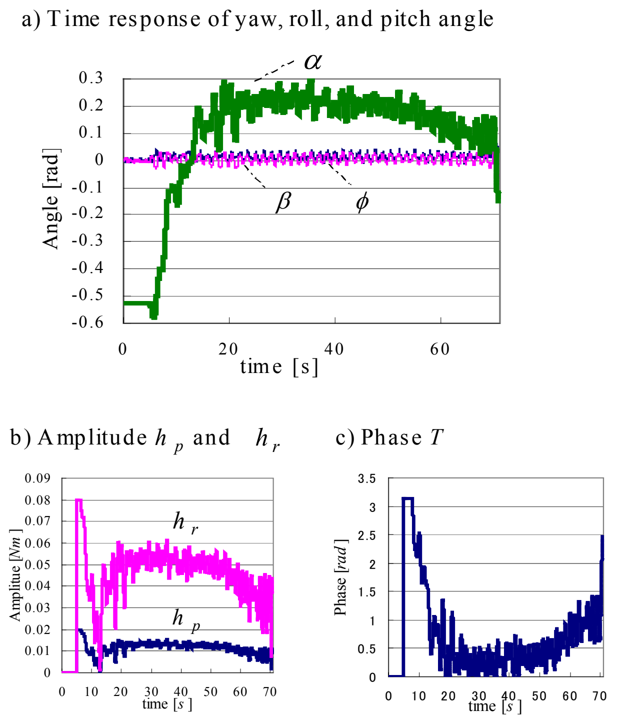

Fig. 14 Time response of unicycle robot controlled with proposed method

Based on these results, the input u_p for the motor that drives the wheel and the input u_r for the motor that drives the disk were determined. We also developed an observer for estimating the posture of the robot using three gyro sensors and two accelerometers.

The experimental results suggest the possibility that the proposed control method can change the yaw direction of the unicycle robot while stabilizing its posture using only two motors. In the future, we will improve the unicycle robot so as to send the control information wirelessly and will confirm the performance of the control method using the improved robot.

VI. Conclusions

In the present paper, we have proposed a control method for changing the yaw direction of the underactuated unicycle robot while stabilizing its posture. Operating inputs consist of a state feedback controller for stabilizing the posture of the robot and sinusoidal inputs for exerting interactions in order to change the yaw direction.

In order to obtain suitable parameters of the sinusoidal input: h_p, h_r, ω, and T, the relationships between the change in the yaw direction and these parameters were examined by simulation, and the following results were obtained:

1. The maximum change was obtained when the phase T = 0 or $T = \pi$ [rad].
2. The yaw angle did not change when T = 1.5 or T = 4.65 [rad].
3. As the amplitude of the sinusoidal input, h_p or h_r increased, the change in the yaw angle increased.
4. The maximum change was obtained when ω = 0.67 [Hz].

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