A Combined QFT/EEAS Design Technique for Uncertain Multivariable Plants

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Abstract **- This paper presents a robust-adaptive control design for uncertain multivariable plants based on Quantitative Feedback Theory (QFT) and Externally Excited Adaptive Systems (EEAS). Design requirements are derived and formulated in terms of different cost functions. Also, a stochastic optimization technique is employed to optimally design the overall robust adaptive controller. This controller can handle large plant parameter uncertainties with lower control gains. Simulation results are provided to show the effectiveness and features of the proposed QFT/EEAS MIMO design methodology compared with the direct MIMO QFT design approach.**

Index Terms – Robust Adaptive Control, QFT, EEAS, Stochastic optimization techniques.

I. Introduction

Quantitative Feedback Theory (QFT) is a powerful tool in the design of robust control systems for uncertain plants. The quantitative approach provides a design methodology which enables the designer to observe clearly the limitations and trade-offs in its design [1].

QFT design methodology for uncertain multivariable plants has provided a number of competing techniques. These can be generalized into two classes, being the non-sequential design methodologies [2] and sequential design methodologies [3, 4].

In solving an $m \times m$ multivariable design problem, the synthesis problem is converted into m equivalent single-loop MISO problems, with parameter uncertainties, external disturbances and performance tolerances are derived from the original problem, and the coupling effects between MISO subsystems are treated as disturbance inputs. These coupling effects need to be rejected in the QFT design of each subsystem. These can be designed by the SISO QFT design technique [5, 6].

The disturbance-rejection requirements will normally dominate the tracking-performance requirements, in the design of each subsystem [7]. On the other hand, in the case of plants with large parameter uncertainties, QFT design technique can lead to controllers with large bandwidth. These can result in high control gains that can cause actuator saturation and reduce the control-loop performance as well as leading to over design.

To overcome these deficiencies, QFT has been combined with other control techniques, such as the eigenstructure assignment [7] and the dynamics inversion technique [8]. Effort has also been put into designing non-diagonal controllers to reduce the gains of controllers [9-11].

Also, an adaptive QFT approach was proposed by [12] using Self Oscillating Adaptive Systems (SOAS). The resulted methodology was insensitive to large gain variations. This however causes limit cycles in the closed loop plant, which is not desirable in many practical applications.

In another approach Externally Excited Adaptive Systems (EEAS) are proposed to replace SOAS [13]. This will circumvent the need for the oscillation condition and improves the quality of closed loop performance.

This paper extends the robust adaptive design of controller using the QFT/EEAS approach [14] to multi-input multi-output (MIMO) systems. In all the quantitative designs, a time-consuming trial-and-error procedure is adapted and a successful compromise between various design requirements is very much dependent on the designer experience. In order to further improve the controller performance, the design steps involved are stated as design objectives and the problem is formulated as a constrained nonlinear optimization problem. The optimization problem is then solved using any of the standard random optimization techniques [15]. Genetic Algorithm is utilized here. Finally, to assess the feasibility and performance of the proposed design approach, simulation results are employed and comparison results with direct MIMO QFT are given which clearly indicate the advantages gained by the proposed design.

II. Quantitative Feedback Theory

QFT is a well established methodology for the design of two-degrees of freedom (2DOF) robust control systems [1]. The QFT method takes into account quantitative information on the plant's variability (uncertainty), the robust stability requirements, tracking-performance specifications, the expected disturbance amplitude and its attenuation requirements. The main steps involved in the QFT design can be summarized as: template generation, loop shaping and pre-filter design. These steps are formulated in an optimal framework and solved using stochastic optimization techniques in [15].

III. EXTERNALLY EXCITED ADAPTIVE SYSTEMS

EEAS is an adaptive control methodology as shown in Fig.1 for single-input single-output (SISO) systems. P represents the plant with varying parameters, while F, G_1, G_2, G_3 are linear compensators, whose values are to be chosen.

The main features of this strategy are: [13]

1- It consists of a two-degrees-of freedom feedback structure with linear elements, except for one nonlinear element N , which is saturated hard by an external periodic signal.

2- If a fast and large (relative to control signals frequencies and amplitudes) periodic signal is injected which saturates N twice per period, then the system response to the control signals is approximately linear.

3- It exhibits the valuable property of zero sensitivity to changes in the plant high-frequency gain factor K, (P(s) = KP_h(s)). This suggests that EEAS is much superior to the linear feedback system.

Quasilinear Representation

The quasilinear properties of EEAS permit the extension of the quantitative linear feedback theory to this system.

Let the input to nonlinear element N (Fig. 1) be $x = A\sin(\omega_o t+\theta)+\sum_i B_i(t) \overset{\Delta}{=} x_o + x_f$ with independent terms. Under certain conditions the output is closely approximated by: [13]

$$y = N_o A\sin(\omega_o t+\theta) + N_f \sum_i B_i(t) \overset{\Delta}{=} y_o + y_f$$
$$N_f = 0.5 N_o \quad , \quad N_o = M/A \quad , \quad M = Cte. \qquad (1)$$

These conditions are: [13]

$$\max_{d,r,P,t} \left| x_f(t) \right| \le A_1/\alpha$$
$$(\omega_{dx}, \omega_{rx})_{\max} \le \beta\omega_o \qquad (2)$$

where, ω_{ix} is the bandwidth of x_f component due to input *i*. (x_r, x_d are forced components due to command input and disturbance signal *r,d*. Also, x_η is due to sensor noise η(t)). $A_1 = \min A$ and α, β are constants depending on the acceptable accuracy; $\alpha = \beta^{-1} = 3$ is suggested in [13]. For example, if N is an ideal relay with output $\pm M_0$ and B_1 is constant, $B_2(t) = B_2 \sin\gamma(t)$ then the error in (1) is 5-10 percent, if $B_1, B_2 < A/3$ and $\gamma < \omega_o/3$; also in (1), $M = 4M_0/\pi$.

In view of (1), if the quasilinear conditions are satisfied, then

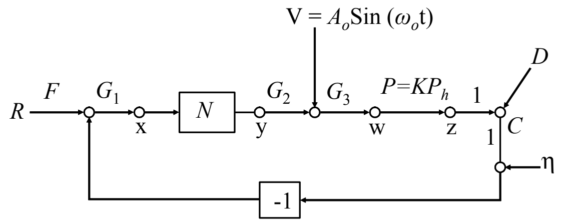

Fig. 1. A canonic EEAS structure [$x = A\sin(\omega_o t+\theta) + x_r + x_d + x_\eta$] , $y \approx (M/A)[x = A\sin(\omega_o t+\theta) + 0.5(x_r + x_d + x_\eta)]$

$$\frac{C}{R} \overset{\Delta}{=} T_r = F\frac{L_f}{1+L_f} \quad , \quad L_f = G_1 G_2 G_3 N_f P = 0.5 L_o \qquad (3)$$

where, L_o and L_f are the transmission loop gains for x_o and x_f , respectively. It is shown that, the output is insensitive to plant gain variations [13].

To employ an EEAS design, the following constraints and cost functions are introduced:

1- To quench any self-induced limit cycle, if N is an ideal relay, $Arg\, L_o(j\omega_\pi) = -180^\circ$, then no limit cycle will exist at ω_π if: [13]

$$\left| L_o(j\omega_\pi) \right| = \frac{M}{A}\left| G_1 G_2 G_3 K P_h(j\omega_\pi) \right| < \rho,$$
(= 1.17 for ideal relay)
$$w_1 = 1.17 - \left| L_o(j\omega_\pi) \right| > 0 \qquad (4)$$

2- To limit $\left| C(j\omega_o) \right|$ to an acceptable value m ; with $(K_{\max} \overset{\Delta}{=} K_2)$

$$\left| C(j\omega_o) \right|_{\max} = \max_K \frac{A_o \left| G_3 K P_h(j\omega_o) \right|}{\left| 1 + L_o(j\omega_o) \right|} = \frac{A_o K_2 \left| G_3 P_h(j\omega_o) \right|}{\left| 1 + L_o(j\omega_o) \right|} \le m,$$
$$w_2 = m - \left| C(j\omega_o) \right|_{\max} \ge 0. \qquad (5)$$

3- To satisfy the quasilinear conditions and disturbance rejection; [13]

$$w_3 = \left| L_o(j\omega_o) \right| - \frac{2\alpha K_2}{m K_1}\left| (b + j\omega_o) Z_e(j\omega_o) \right| \ge 0 \qquad (6)$$

$$w_4 = \left| 1 + L_f(j\omega_o) \right| - \frac{\alpha K_2}{m K_1}\left| (b + j\omega_o) D(j\omega_o) \right| \ge 0 \qquad (7)$$

where, Z_e is assumed to be the extreme plant output, $X_e = Z_e / G_3 G_2 K_1 P_h N_f = qb/(s+b)$ is chosen to be the model of extreme value of $x_f(t)$ in the ω domain, such that $qb \le A_1/\alpha$ [from (2)], this choice is justified in [13], the plant high-frequency gain K belongs to $[K_1, K_2]$.

An optimal quantitative synthesis procedure is addressed for the EEAS [14], permitting systematic optimum design to satisfy numerical specifications in the face of given ranges of plant parameters variations.

IV. DIRECT MIMO QFT DESIGN

The MIMO QFT design technique provides a design procedure to synthesize a fixed diagonal controller transfer function matrix *G*(s) and prefilter *F*(s) to satisfy specifications on the closed-loop system shown in Fig.2, where *P*(s) is the MIMO uncertain plant.

The basic principle for MIMO QFT design is to convert the MIMO control system into a set of equivalent MISO control systems. Using fixed-point theorem, the MIMO control problem for an *m* × *m* system can be converted into *m* equivalent single-loop MISO problems, each with two inputs and one output. The objective of the design is to achieve set point tracking, while minimizing the outputs due to the disturbance inputs (cross-coupling effects) [5, 6].

V. COMBINED QFT/EEAS DESIGN

A. Problem formulation

With no loss of generality and for simplicity, the design process is developed for 2 × 2 MIMO plants. The procedure can be easily extended to the general MIMO case. Consider the feedback structure shown in Fig. 3. The transfer function matrix $P(s) = [p_{ij}(s)]$, $i, j = 1,2$ represents the LTI uncertain 2 × 2 MIMO plant to be controlled. The $G_i = diag[g_{i1}(s), g_{i2}(s)]$, $i = 1,2,3$ and $F(s) = diag\ [f_{11}(s), f_{22}(s)]$, which are assumed diagonal, represent the feedforward compensators and the prefilter matrix, respectively. Also, the nonlinear elements N_1 and N_2 are assumed to be ideal relays with outputs $\pm M_{01}$ and $\pm M_{02}$. Moreover, V_1 and V_2 are the excitation signals.

Let $T_{Y/R}(s)$ be the input-output relation from the input $R(s)$ to the output $Y(s)$, which is clearly derived as

$$T_{Y/R}(s) = [I + P(s)G(s)]^{-1}\ P(s)G(s)F(s) \tag{8}$$

where, $G = G_3 G_2 N_f G_1$ ($N_f = diag\ (n_{f1}, n_{f2})$, are the describing functions of the relays).

Due to uncertainty, $P \in \{\mathrm{P}\}$ is a set of possible plants and it is assumed here that the plant set is finite or can be adequately approximated by a finite set so that numerical algorithms can be developed. The combined QFT/EEAS control design task is to find $G_i(s)$ and $F(s)$ with proper rational and stable elements, in order to satisfy the performance specifications $\forall P \in \{\mathrm{P}\}$. For example, tracking specifications may require that $\forall P \in \{\mathrm{P}\}$,

$$B_{ij}(\omega) \le \left|T_{Y/R}(j\omega)\right|_{ij} \le A_{ij}(\omega) \tag{9}$$

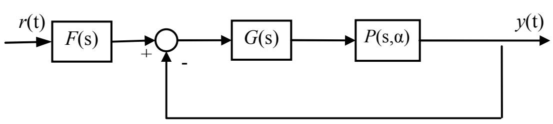

Fig. 2. 2DOF MIMO-QFT Control Structure

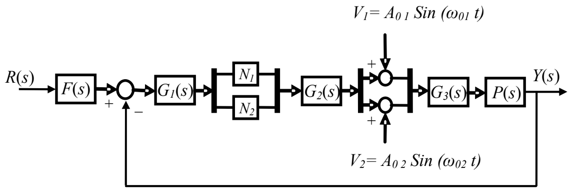

Fig. 3. The combined QFT/EEAS structure for 2 × 2 MIMO plants

where, $A_{ij}(\omega)$ and $B_{ij}(\omega)$ are the upper and lower specifications. For simplicity, this paper will concentrate on tracking performance in (9), but there may be other specifications on sensitivity, sensor noise to input sensitivity, as well as engineering considerations such as those in direct MIMO-QFT. At high frequencies the benefits of feedback are negligible. High frequency specifications will result in large bandwidth with very little closed-loop performance improvement. It is thus recommended that the specifications to be enforced to the lowest possible frequency ω_h (the Horowitz frequency). In addition, an implicit design objective is the minimization of the loop bandwidths when sensor noise attenuation is concerned.

B. Development of the Design Process

In Fig.3, if the quasilinear conditions are satisfied, the closed-loop transfer of this MIMO control system could be expressed as

$$Y(s) = T_{Y/R}\ R(s) = \begin{bmatrix} t_{11}(s) & t_{12}(s) \\ t_{21}(s) & t_{22}(s) \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \tag{10}$$

where, *Y*(s) is the system output, *R*(s) is the input vector.

Through an appropriate transformation, the four transfer functions in the closed-loop transfer matrix are derived as:

$$\text{For input } r_1: \ t_{11} = \frac{g_1 q_{11} f_{11} - \left(\dfrac{t_{21}}{q_{12}}\right) q_{11}}{1 + g_1 q_{11}}, \quad t_{21} = \frac{-\left(\dfrac{t_{11}}{q_{21}}\right) q_{22}}{1 + g_2 q_{22}}$$

$$\text{For input } r_2: \ t_{12} = \frac{-\left(\dfrac{t_{22}}{q_{12}}\right) q_{11}}{1 + g_1 q_{11}}, \quad t_{21} = \frac{g_2 q_{22} f_{22} - \left(\dfrac{t_{12}}{q_{21}}\right) q_{22}}{1 + g_2 q_{22}} \tag{11}$$

where, $q_{11} = \Delta/p_{22}$, $q_{12} = -\Delta/p_{12}$, $q_{21} = -\Delta/p_{21}$, $q_{22} = \Delta/p_{11}$, $\Delta = p_{11}\,p_{22} - p_{12}\,p_{21}$ and $g_i = g_{3i}\, g_{2i}\, n_{fi}\, g_{1i}$, $i = 1,2$.

These four transfer functions represent the resulting four equivalent single-loop MISO control systems, as expressed by their signal flow graphs shown in Fig. 4.

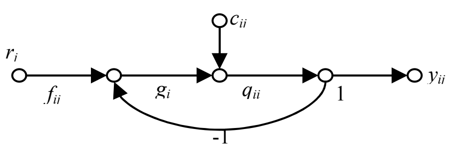

Fig. 4. 2 × 2 MISO Structure for t_{ij} ($f_{ij} = 0$ for $i \ne j$, $c_{ij} = -\sum_{k \ne i} \dfrac{t_{kj}}{q_{ik}}$)

It follows that only the two systems on the diagonal line have both input tracking and disturbance rejection requirements. The other two systems have only disturbance rejection requirement. Furthermore, the two systems of the same rank (e.g. t_{11} and t_{12}) have the same subplant (q_{11}) and compensator (g_1), i.e. the same dynamic feedback control loop. They can be designed together by combining their external disturbance inputs. Thus, the design of the 2 × 2 MIMO plant is further simplified into the design of two equivalent MISO systems, which can be designed by the SISO QFT/EEAS approach [14]. The various steps of the combined design technique are formulated in terms of appropriate cost functions and respective algebraic constraint [14, 15]. The resulting nonlinear multi-objective constrained optimization problem can easily be solved using any of the stochastic optimization techniques.

VI. Design Example

In this section, simulation results are used to illustrate the proposed design method. Consider the following 2 × 2 MIMO plant with transfer function matrix [2], [15]:

$$P(s) = \begin{bmatrix} \dfrac{k_{11}}{1+s\,A_{11}} & \dfrac{k_{12}}{1+s\,A_{12}} \\ \dfrac{k_{21}}{1+s\,A_{21}} & \dfrac{k_{22}}{1+s\,A_{22}} \end{bmatrix}$$

and a total of nine plant cases given in Table I. The first plant is taken as the nominal.

Specifications (for all plants)

(1) The tracking specifications, $B_{ij} \le \left| T_{Y/R}(j\omega) \right|_{ij} \le A_{ij}$ are basically non-interacting, and are enforced to $\omega = 10$ rad/sec.

On-diagonal:

$$A_{ii}(\omega) = \left| \frac{25}{s^2 + 6s + 25} \right|_{s=j\omega} \quad and \quad B_{ii}(\omega) = \left| \frac{4}{s^2 + 4.4s + 4} \right|_{s=j\omega}$$

Off-diagonal:

$$A_{ij}(\omega) = 0.1 \quad and \quad B_{ij} = 0$$

(2) Stability margin: $\left| 1/(1+L_{f_i}) \right| \le 3.5$ dB for all plants.

where, $L_{f_i} = g_{3i}\, g_{2i}\, n_{fi}\, g_{1i}\, q_{ii}$ and i corresponds to the i^{th} loop and this would indicate a gain margin and phase margin of 9.6dB and 39deg respectively.

The process of the nonlinear optimization problem results in the following optimal robust controllers: ($m = 0.1$, $b = 1$, $\alpha = \beta^{-1} = 3$).

$$g_{11}(s) = \frac{2.59\times10^2}{(s+1813)}, g_{21}(s) = \frac{4\times10^4(s+1181)}{s(s+3.94\times10^4)}, g_{31}(s) = \frac{0.12}{(s+3.02\times10^4)}$$

$$g_{12}(s) = \frac{22.41}{(s+6142)}, g_{22}(s) = \frac{5.9\times10^5(s+1276)}{s(s+5.76\times10^4)}, g_{32}(s) = \frac{0.2}{(s+5.3\times10^4)}$$

Also, the other design parameters are as follows:

$$\omega_{0_1} = 51.2\ ,\ A_{0_1} = 4.2\times10^3, M_{0_1} = 310.6, A_{\min_1} = 1.4\times10^{-4}$$

$$\omega_{0_2} = 39.0\ ,\ A_{0_2} = 1.9\times10^4, M_{0_2} = 66.5\ , A_{\min_2} = 1.8\times10^{-5}$$

And, the designed pre-filters are

$$f_{11}(s) = \frac{2.73(s+359.2)}{(s+498)(s+1.97)}\ ,\ f_{22}(s) = \frac{7.88(s+72.83)}{(s+498.9)(s+1.15)}$$

Fig.5, Fig.6 and Fig.7 show the performance of the closed-loop system in tracking step commands, for different plant conditions.

It is obvious that the combined robust adaptive strategy has reached the desired performance with lower loop bandwidths. The crossover frequency of the first loop gain for all the uncertainty range is $\omega_{c_1} = 29.63$ rad/sec, while in an expert design, given by Horowitz [2], the loop crossover frequency varies between 9.96 rad/sec and 205 rad/sec, also in the optimal design of [15], this belongs to $[6.28\ ,\ 102.7]$ rad/sec. Also, in the second loop the cut off frequency, for all nine plant cases is $\omega_{c_2} = 19.71$ rad/sec, while in Horowitz's design it belongs to [7.31, 54.5] rad/sec, and in the optimized design of [15] it varies between 6.42 rad/sec and 75.8 rad/sec.

TABLE I
Plant Conditions used in Example

No.	k_{11}	k_{22}	k_{12}	k_{21}	A_{11}	A_{22}	A_{12}	A_{21}
1	1	2	0.5	1	1	2	2	3
2	1	2	0.5	1	0.5	1	1	2
3	1	2	0.5	1	0.2	0.4	0.5	1
4	4	5	1	2	1	2	2	3
5	4	5	1	2	0.5	1	1	2
6	4	5	1	2	0.2	0.4	0.5	1
7	10	8	2	4	1	2	2	3
8	10	8	2	4	0.5	1	1	2
9	10	8	2	4	0.2	0.4	0.5	1

VII. Conclusion

Combined QFT/EEAS design methodology was extended to MIMO control systems. This method provides an alternative technique to handle MIMO robust-adaptive control system design based on the MIMO-QFT design technique and Externally Excited Adaptive Systems (EEAS). Proposed design technique results in controllers of acceptable low loop bandwidth for highly uncertain MIMO plants, by reducing over design compared with the direct MIMO-QFT (using a diagonal controller only). To facilitate this complex design procedure, relevant cost functions and constraints were introduced, and the problem was formulated as a nonlinear optimization problem. This problem was then solved using a stochastic optimization technique that results in an optimal design. Simulation results were provided to show the feasibility and effectiveness of the proposed design methodology.

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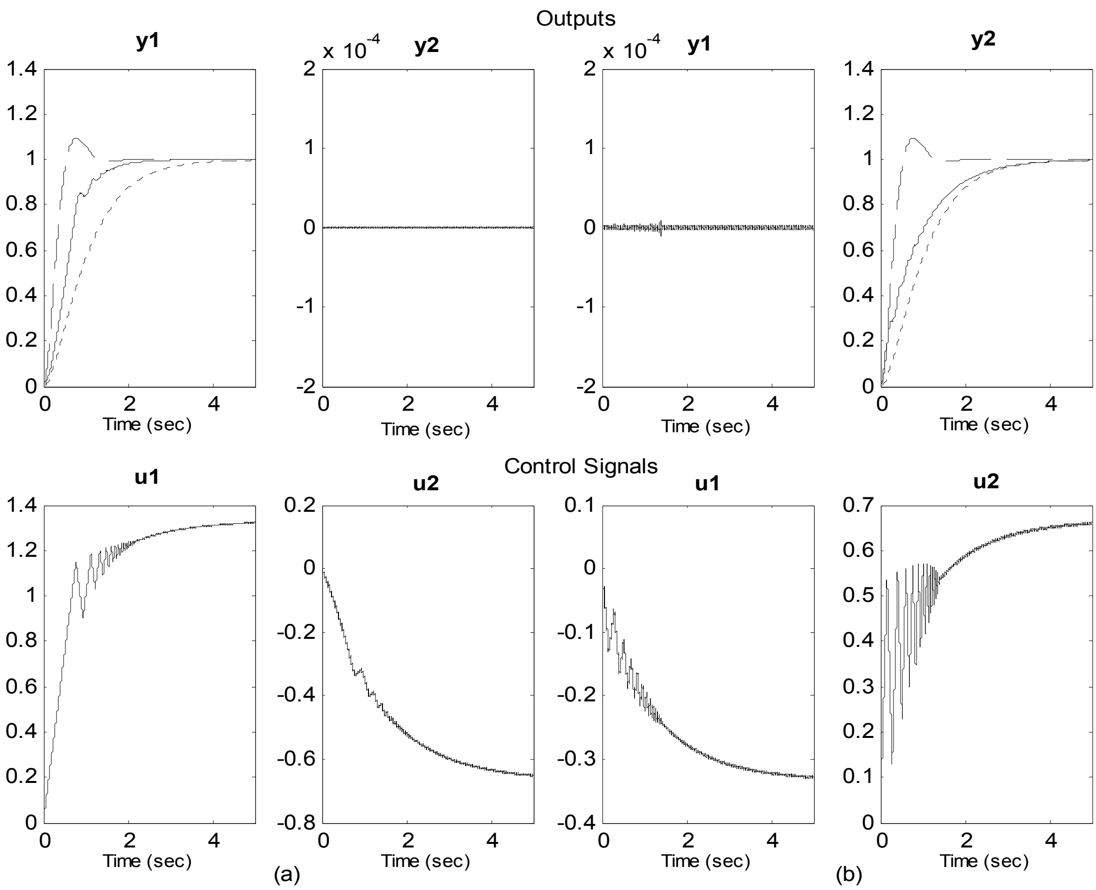

Fig. 5. Simulation results for plant parameters as in case3, (a) the command input is $R=\begin{bmatrix}1 & 0\end{bmatrix}^T$ (b) the command input is $R=\begin{bmatrix}0 & 1\end{bmatrix}^T$.

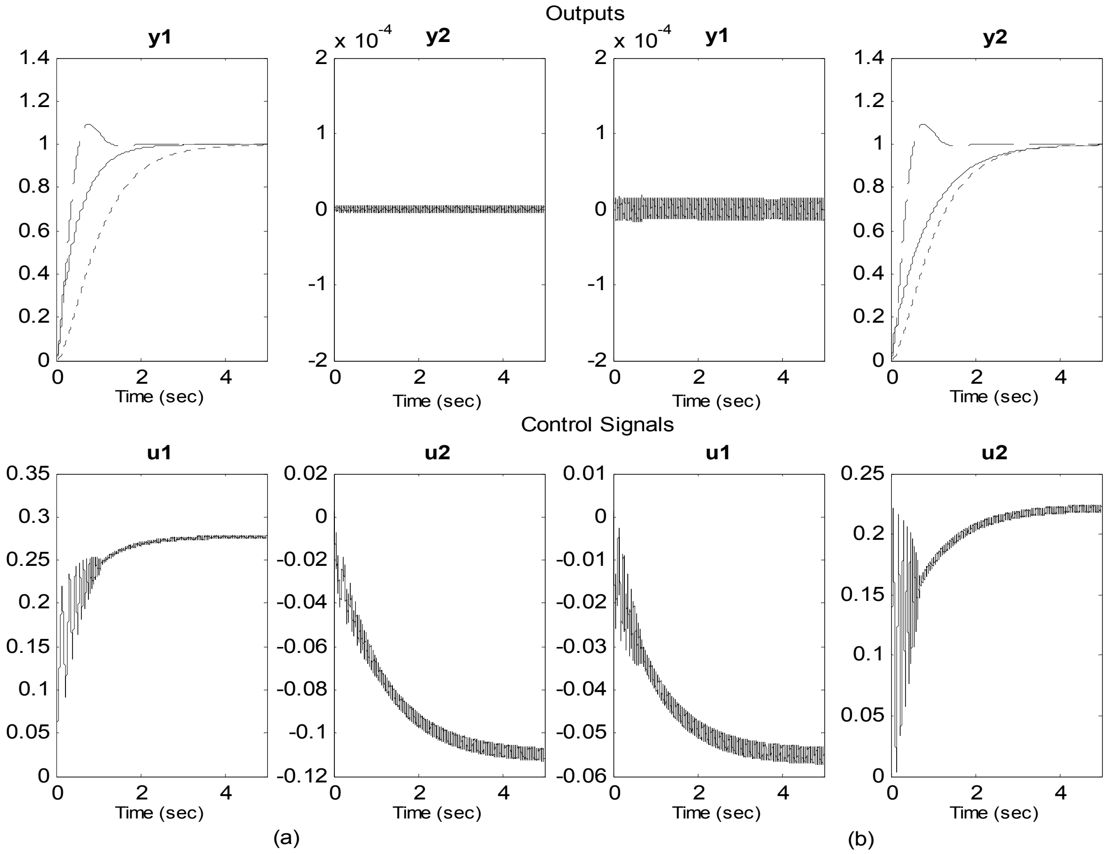

Fig. 6. Simulation results for plant parameters as in case6, (a) the command input is $R = [1 \quad 0]^T$ (b) the command input is $R = [0 \quad 1]^T$.

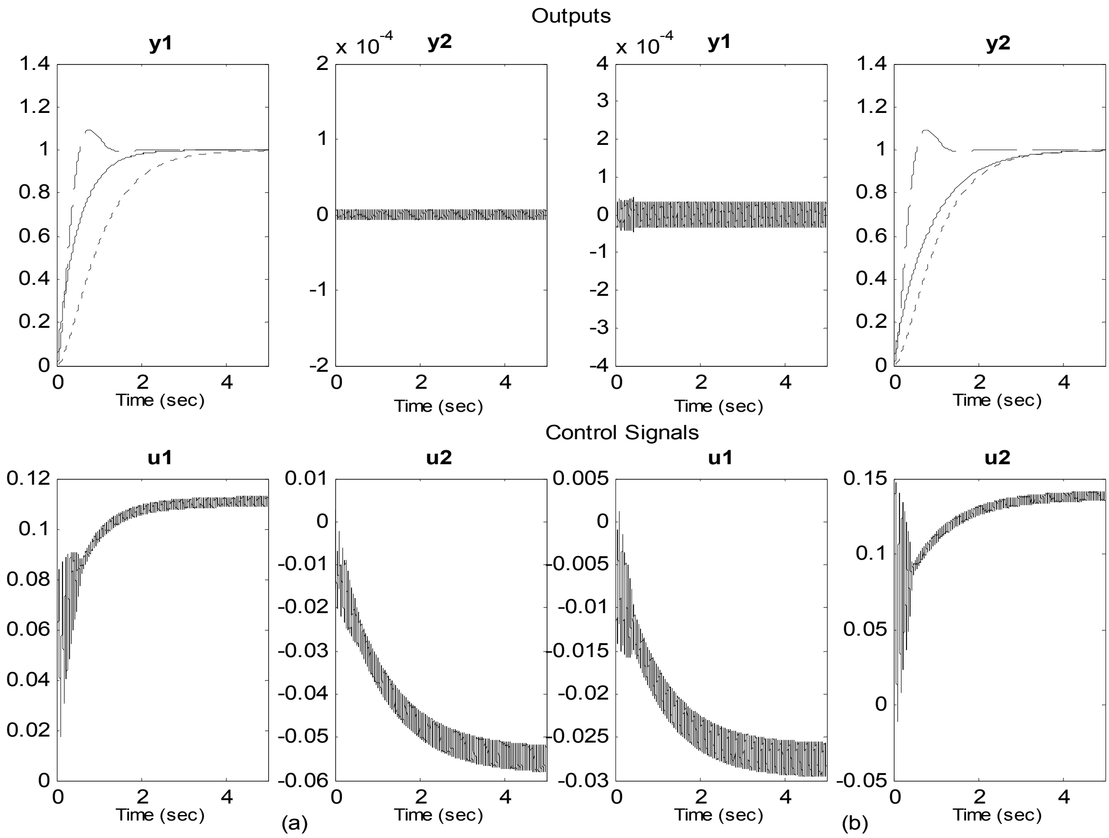

Fig. 7. Simulation results for plant parameters as in case9, (a) the command input is $R = [1 \quad 0]^T$ (b) the command input is $R = [0 \quad 1]^T$.