Robust Optimal Tracking Control of Hybrid Systems: Based on Dynamical Programming

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***Abstract*— This paper studies the optimal tracking problem for LTI switched system.A linear quadratic error criterion is used as the cost function.Firstly,Based on the dynamical programming, the optimal tracking algorithm is designed.Secondly, the stability analysis of this feedback optimal tracking problem is discussed.Then,faced with nonlinear uncertainty of statistics in dynamic switched systems the robust stability condition of linear-quadratic optimal tracking problems is obtained .At last, a numerical example shows the main algorithm we designed.**

***Index Terms*— Hybrid system, switched system, linear quadratic optimization, tracking, robust control.**

I. INTRODUCTION

The optimal problems for hybrid systems have attracted many researchers in recent years[15-18].This paper studies the optimal tracking problem for linear time invariant(LTI)switched systems.the cost function we used is the linear quadratic error criterion.Optimal tracking problem is a well-studied topic for single-input-single-output(SISO)systems[1], and multiple-input-multiple-output(MIMO)systems[2-4]. More recently, there has also been largely concentrated on single-input-two-output(SITO)systems. For example, [5-6]considered "algebraic" design tradeoffs between feedback properties examined at different loop-breaking points, in which the analysis is focused on a single frequency. [7]studied Bode integral relations and an optimal tracking problem for SITO systems. For general nonright-invertible system, [8]discussed the problem of cheap control performance.

It is well known that the most practical systems are hybrid in nature. These systems consist of a continuous time system and a discrete event system that interact and influence each other. there are many different models for hybrid systems.Several modelling frameworks can be broadly divided into two groups:Those that extend the traditional event-driven models to include time-driven models;and those time-driven ones to include event-driven models.Examples from the former group include Petri net models to allow state transition times to be determined by time-driven dynamics as in[9].Examples from the latter group include time-driven models are shown in[10-11]where discrete-events are injected as jump processes into a time-driven model.Switched systems,tend to be described by similar models

This paper discusses the optimal tracking problem of the linear time-invariant switched system, a class of hybrid systems consists of nonzeno sequences. Here, we propose a new method called dynamical programming, combining the idea of discrete dynamical programming with continuous infinite and finite time quadratic optimal track.

The structure of the paper is organized as follows. In section2, the model of switched systems and optimal tracking problem are described. In section 3, the algorithm of quadratic optimal tracking problem is designed based on dynamical programming. Then, the stability of the designed optimal tracking closed-loop system is discussed in section 4. Then,faced with nonlinear uncertainty of statistics in dynamic switched systems the robust stability condition of linear-quadratic optimal tracking problems is obtained in section 5. Section 6 shows the numerical example with satisfactory main result.

II. PROBLEM FORMULATION

In this paper, we consider switched systems of the form:

$$\dot{x} = f_i(x,u) = A_i x + B_i u$$
$$y = C_i x, i = 1,2,\cdots,K+1 \tag{1}$$

where $x \in R^n, u \in R^m$.Now we assume that A_i, B_i is is stabilizable, and A_i, C_i is observable.y is the output of (1)which is also the tracking signal.

A switching sequence in $t \in [t_0, t_f]$ regulates the sequence of active subsystems and is defined as

$$\sigma = (t_0, e_0), (t_1, e_1), \cdots, (t_K, e_K) \tag{2}$$

where$t_0 \leq t_1 \leq t_2 \leq \cdots \leq t_K \leq t_f$.During the time interval $[t_i, t_{i+1})$ subsystem i is active. In our study, we only consider nonzeno sequence which switch at most a finite number of times in $[t_0, t_f)$.

Then, we assume that the linear time-invariant switched system as followed:

$$\dot{z} = F_i z$$
$$\tilde{y} = H_i z, i = 1,2,\cdots,K+1 \tag{3}$$

The output of (3) is$\tilde{y}$, which is tracked by$y$.

So the optimal tracking problem is to find an optimal input $u^*(\cdot)$ and the switching instants in order to minimize the cost function:

$$J = x^T(t_f)Fx(t_f) + \frac{1}{2}\int_{t_0}^{t_f} \left[(y-\tilde{y})^T Q (y-\tilde{y}) + u^T R u\right] dt \tag{4}$$

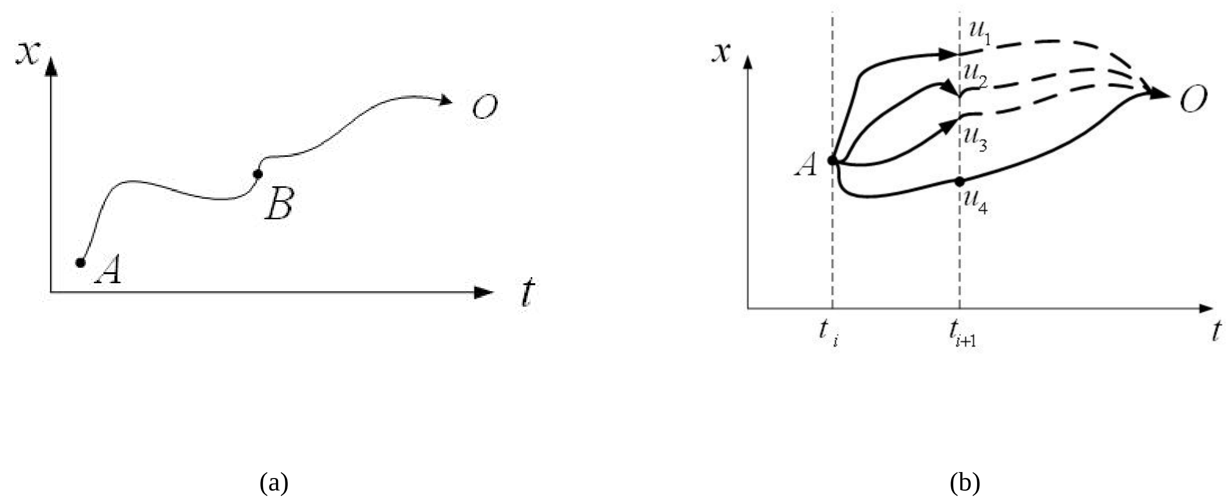


Fig. 1. Principle of Optimality.

Where F, Q, R are symmetric positive semi-definite matrices.

III. LINEAR QUADRATIC OPTIMAL CONTROL OF SWITCHED SYSTEM BASED ON DYNAMIC PROGRAMMING

A. Dynamic Programming

Dynamic Programming is a fundamental method to solve optimal control of discrete system, it can also be used in continuous system. Branicky used it to hybrid systems [13]. This method we use here is to solve the computing. If the system has$K+1$ different subsystems, and the switching times is K ,then the number of total possible paths is $(K+1)^{K-1}$, so the computing is $K \times (K+1)^{K-1}$.

In fact, there are lots of paths don't exist subject to the constraints of continuous and discrete condition. So, if we find out a method to list out only the possible paths, then the whole computing will be reduced greatly.

The term Dynamic Programming we use here was introduced with Bellman's Principle of Optimality [12]. We use the principle of Optimality here simply states that if one step is taken along an optimal path from A to O , then the remaining path is also optimal for the new point B . See Fig.1(a). The main use of the Principle of Optimality is that we know all optimal paths for any initial state$x$ at time t_{i+1} , then every optimal path staring at time t_i must use one of these optimal paths from time t_{i+1} and onwards. See Fig.1(b). By using this, the computing is reduce.

Now, we use this to design an algorithm to solve the optimization problem.

B. The algorithm of optimal control of switched system

Now we define:

$$\bar{x} = \begin{pmatrix} x \\ z \end{pmatrix}, \bar{A}_i = \begin{pmatrix} A_i & 0 \\ 0 & F \end{pmatrix}, \bar{B}_i = \begin{pmatrix} B_i \\ 0 \end{pmatrix}$$

$$\bar{Q}_i = \begin{pmatrix} C_i^T Q C_i & -C_i^T Q H_i \\ -H_i^T Q C_i & H_i^T Q H_i \end{pmatrix}, \bar{R} = R$$

And the problem is changed into: Based on:

$$\dot{\bar{x}} = \bar{A}_i \bar{x} + \bar{B}_i u, i = 1, 2, \cdots, K+1 \tag{5}$$

Find an optimal input$u^*(\cdot)$ in order to minimize the cost function:

$$J = x^T(t_f) F x(t_f) + \frac{1}{2} \int_{t_0}^{t_f} \left(\bar{x}^T \bar{Q}_i \bar{x} + u^T \bar{R} u \right) dt \tag{6}$$

where $\bar{Q}_i \geq 0, \bar{R} = R > 0$ if $Q \geq 0$ and A_i, C_i, F, H_i is observable.

Now we give a specific algorithm of optimal tracking control of switched systems:

First step: At first, we introduce a finite time quadratic performance of the continuous system:

$$\dot{\bar{x}} = \bar{A}_{K+1} \bar{x} + \bar{B}_{K+1} u \tag{7}$$

at$[t_K, t_f]$,and find optimal input in order to minimize the cost function:

$$J_{K+1} = x^T(t_f) F x(t_f) + \frac{1}{2} \int_{t_K}^{t_f} \left(\bar{x}^T \bar{Q}_i \bar{x} + u^T \bar{R} u \right) dt \tag{8}$$

Where F, Q and R are real, symmetric and positive definite matrices. According to the optimal control law of the quadratic optimal control problem, J_{K+1} can be minimized from the positive definite solution $P(t)$ to the algebraic Riccati equation:

$$\dot{\bar{P}}(t) = -\bar{P}(t)\bar{A}_{K+1} - \bar{A}_{K+1}^{T}\bar{P}(t) +$$
$$\bar{P}(t)\bar{B}_{K+1}\bar{R}^{-1}\bar{B}_{K+1}^{T}\bar{P}(t) - \bar{Q}, P(t_f) = F \tag{9}$$

and the corresponding optimal linear state feedback control law is

$$u(t) = -\bar{R}^{-1}\bar{B}_{K+1}^{-1}\bar{P}_{K+1}(t)\bar{x}(t) \tag{10}$$

So the optimal performance can also be easily computed as

$$J_{K+1}^{*}=\frac{1}{2}\bar{x}^{T}(t_K)\bar{P}_{K+1}(t_K)\bar{x}(t_K) \tag{11}$$

Second step:at $[t_{K-1},t_K]$,for the corresponding continuous dynamical subsystem:

$$\dot{\bar{x}}=\bar{A}_K\bar{x}+\bar{B}_K u \tag{12}$$

We introduce the finite time quadratic performance of the continuous systems:

$$J=J_{K+1}^{*}+\frac{1}{2}\int_{t_{K-1}}^{t_K}\left(\bar{x}^{T}\bar{Q}_i\bar{x}+u^{T}\bar{R}u\right)dt \tag{13}$$

Where J_{K+1}^{*} is the minimized J_{K+1} above. And the corresponding optimal linear state feedback control law is:

$$u(t)=-\bar{R}^{-1}\bar{B}_K^{-1}\bar{P}_K(t)\bar{x}(t) \tag{14}$$

where$\bar{P}(t)$ satisfied:

$$\dot{\bar{P}}(t)=-\bar{P}(t)\bar{A}_K-\bar{A}_K^{T}\bar{P}(t)+\bar{P}(t)\bar{B}_K\bar{R}^{-1}\bar{B}_K^{T}\bar{P}(t)-\bar{Q}$$
$$\bar{P}(t_K)=\bar{P}_{K+1}(t_K) \tag{15}$$

So we get the optimal performance as:

$$J_K^{*}=\frac{1}{2}\bar{x}^{T}(t_{K-1})\bar{P}_K(t_{K-1})\bar{x}(t_{K-1}) \tag{16}$$

which is the function of t_{K-1},t_K .

From the step above, we get the optimal input of the switched system as:

$$u(t)=-\bar{R}^{-1}\bar{B}_i^{-1}\bar{P}_i(t)\bar{x}(t),t\in[t_{i-1},t_i],i=1,2,\cdots K+1 \tag{17}$$

And the optimal performance computed as:

$$J^{*}=\frac{1}{2}\bar{x}^{T}(t_0)\bar{P}_1(t_0)\bar{x}(t_0) \tag{18}$$

which is the function of $t_1,t_2,\cdots,t_K$.

now we define

$$\bar{P}_i(t)=\begin{pmatrix} P_i(t) & P_{i,12}(t) \\ P_{i,12}^{T}(t) & P_{i,22}(t), \end{pmatrix} \tag{19}$$

So we can get:

$$-P_i(t)A_i-A_i^{T}P_i(t)-C_i^{T}QC_i+P_i(t)B_iR^{-1}B_i^{T}P_i(t)$$
$$=\dot{P}_i(t) \tag{20}$$

$$-P_{i,12}(t)F_i-A_i^{T}P_{i,12}(t)+C_i^{T}QH_i+P_i(t)B_iR^{-1}B_i^{T}P_{i,12}(t)$$
$$=\dot{P}_{i,12}(t) \tag{21}$$

$$-P_{i,22}(t)F_i-F_i-F_i^{T}P_{i,22}(t)-H_i^{T}QH_i-P_{i,12}^{T}(t)B_iR^{-1}B_i^{T}$$
$$P_{i,12}(t)=\dot{P}_{i,22}(t) \tag{22}$$

and we can solve $P_i(t),P_{i,12}(t),P_{i,22}(t)$ based on (20),(21),(22).

So we can give the optimal continuous control $u^{*}(t)$ of problem (1)-(4).

$$u^{*}(t)=-R^{-1}\cdot\begin{pmatrix} B_i^{T} & 0 \end{pmatrix}\cdot\begin{pmatrix} P_i(t) & P_{i,12}(t) \\ P_{i,12}^{T}(t) & P_{i,22}(t) \end{pmatrix}\cdot$$

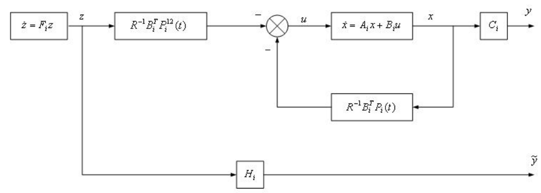


Fig. 2. Designation of tracking structure.

$$\begin{pmatrix} x(t) \\ z(t) \end{pmatrix}$$
$$=-R^{-1}B_i^{T}P_i(t)x(t)-R^{-1}B_i^{T}P_{i,12}(t)z(t) \tag{23}$$

and also get the optimal performance as:

$$J^{*}=\frac{1}{2}\cdot\begin{pmatrix} x^{T}(t_0) & z^{T}(t_0) \end{pmatrix}\cdot\begin{pmatrix} P_1(t_0) & P_{1,12}(t_0) \\ P_{1,12}^{T}(t_0) & P_{1,22}(t_0) \end{pmatrix}\cdot$$
$$\begin{pmatrix} x(t_0) \\ z(t_0) \end{pmatrix}$$
$$=\frac{1}{2}x^{T}(t_0)P_1(t_0)x(t_0)+\frac{1}{2}z^{T}(t_0)P_{1,22}(t_0)z(t_0)+$$
$$x^{T}(t_0)P_{1,12}(t_0)z(t_0) \tag{24}$$

In Fig.2,the optimal tracking controller based on the above algorithm for each subsystem of given switched systems has been shown.

IV. THE ANALYSIS OF STABILITY RESULT OF THE OPTIMAL TRACKING CLOSED-LOOP SWITCHED SYSTEMS

Nowadays, many results on the Lyapunov stability of hybrid systems has been studied. Branicky[13]and Pettersson[14]presented multiple Lyapunov functions approaches.Based on the above algorithm, we obtain the optimal feedback controller for the tracking problem.Now our aim is to analyze the stability of this feedback controller.In order to analyze the stability of it, we introduce a lemma as following firstly.

Lemma 4.1.[14]If there exists scalar functions V_q : $I(q)\to R$, each $V_q(x)$ is differentiable in x, $\forall x\in I(q),q\in X_D$, and class K functions $\alpha:R^{+}\to R^{+}$, and $\beta:R^{+}\to R^{+}$ such that

$$\forall x\in I(q),\alpha(\|x\|)\leq V_q(x)\leq\beta(\|x\|),q\in X_D \tag{25}$$
$$\forall x\in I(q),V_q(x)\leq 0,q\in X_D$$

$$\forall x \in I(q) \frown I(r), V_r(x) \leq V_q(x)$$

then the system is stable in the sense of Lyapunov. The main result on the stability of the close-loop system designed by the proposed algorithm can be described as the following theorem.

Theorem 4.2. The closed-loop linear switched system by the dynamic programming algorithm is stable in the sense of Lyapunov.

proof: From the above theorem, we define the Lyapunov's function as:

$$V_i\left[\bar{x}^T(t)\right] = \bar{x}^T(t)\bar{P}_i(t)\bar{x}(t), i = 1, 2, \cdots, K+1$$

If$S \geq 0, \bar{R} > 0, \bar{Q} > 0$, then

$$V_i\left[\bar{x}(t)\right] = \bar{x}^T(t)\bar{P}_i(t)\bar{x}(t) > 0$$

$$\dot{V}_i\left[\bar{x}(t)\right] = \frac{dV\left[\bar{x}(t)\right]}{dt}$$

$$= \dot{\bar{x}}^T(t)\bar{P}_i(t)\bar{x}(t) + \bar{x}^T(t)\dot{\bar{P}}_i(t)\bar{x}(t) + \bar{x}^T(t)\bar{P}_i(t)\dot{\bar{x}}(t)$$

$$= \bar{x}^T(t)\left[\bar{A}_i^T\bar{P}_i(t) - \bar{P}_i\bar{B}_i\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)\right]\bar{x}(t)$$

$$+\bar{x}^T(t)\left[\bar{P}_i(t)\bar{A}_i - \bar{P}_i(t)\bar{B}_i\bar{R}^{-1}B_i^T\bar{\bar{P}}_i(t)\right]\bar{x}(t)$$

$$+\bar{x}^T(t)\left[\dot{\bar{P}}_i(t)\right]\bar{x}(t)$$

$$= -\bar{x}^T(t)\left[\bar{Q} + \bar{P}_i(t)\bar{B}_i\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)\right]\bar{x}(t)$$

$$\leq 0$$

$$V_{i+1}\left[\bar{x}(t_i)\right] - V_i\left[\bar{x}(t_i)\right]$$

$$= \bar{x}^T(t)\bar{P}_{i+1}(t)\bar{x}(t) - \bar{x}^T(t)\bar{P}_i\bar{x}(t)$$

$$= \bar{x}^T(t)\left[\bar{P}_{i+1}(t_i) - \bar{P}_i(t_i)\right]\bar{x}(t)$$

$$= 0$$

and $V_i\left[\bar{x}(t)\right] \to 0$ as $\bar{x} \to \infty$.

So the closed-loop linear switched system are stable in the sense of Lyapunov. In order to show the above statement directly, the common Lyapunov function established by the Dynamic programming can be seen in Fig.3, precisely, it shows the decrease of energy of Lyapunov function in the stability result of linear-quadratic optimization.

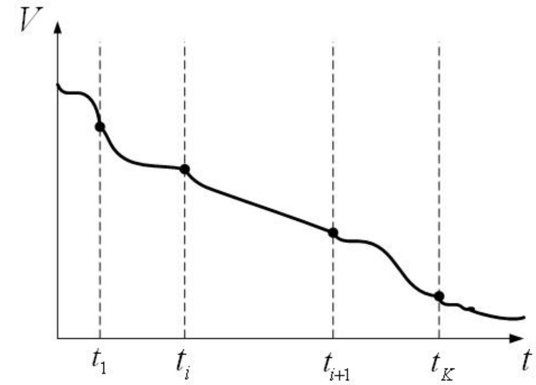


Fig. 3. Illustration of the decrease of energy in the stability result.

V. THE ANALYSIS OF ROBUST STABILITY RESULT OF THE OPTIMAL TRACKING CLOSED-LOOP SWITCHED SYSTEMS

From the above stability analysis, we know that the closed-loop switched systems are stable in the sense of Lyapunov.So in this section, the robust stability of linear-quadratic optimal tracking problems is discussed faces with nonlinear uncertainty of statistics in dynamic switched systems.The purpose is to keep the close-loop switched systems stable in the sense of Lyapunov even if the switched systems consists of nonlinear uncertainty.

We focus on the robust stability against the nonlinear uncertainty as follows :

$$\Theta(\zeta), \zeta = K^*(t)x(t), K^*(t) = \bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t) \tag{26}$$

So the optimal tracking controller is described as:

$$\dot{\bar{x}} = \bar{A}_i\bar{x}(t) - \bar{B}_i\Theta(\zeta), i = 1, 2, \cdots K+1 \tag{27}$$

Now our aim is to find the conditions the nonlinear uncertainty must be satisfied, in order to keep the closed-loop switched systems stable in the sense of Lyapunov.

Theorem 5.1. Faced with linear-quadratic optimal tracking problems for switched systems, suppose the feedback of optimal control of switched system consists of nonlinear uncertainty $\Theta(\zeta)$,if $\Theta(\zeta)$ satisfies the following condition:

$$\zeta^T \cdot \bar{R} \cdot \Theta(\zeta) \geq l \cdot \zeta^T \cdot \bar{R} \cdot \zeta, \forall \zeta \neq 0 \tag{28}$$

Where:

$$\zeta = K^*(t)x(t), K^*(t) = \bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t), i = 1, 2, \cdots K+1$$

and $l = \frac{1}{2} + \varepsilon(t)$, where

$$\varepsilon(t) > 0, \min_{t \to 0^+} = \frac{1}{2}$$

.

proof: From the stable condition of optimal control problem, we get $F \geq 0$,$\bar{R} > 0$,$\bar{Q} > 0$,

So we define the Lyapunov's function as:

$$V_i[\bar{x}(t)] = \bar{x}^T(t)\bar{P}_i(t)\bar{x}(t), i = 1, 2, \cdots K+1$$

if$\bar{F} \geq 0$,$\bar{R} > 0$,$\bar{Q} > 0$,

Then

$$V_i[\bar{x}(t)] = \bar{x}^T(t)\bar{P}_i(t)\bar{x}(t) > 0$$

$$\dot{V}_i[\bar{x}(t)] = \frac{dV[\bar{x}(t)]}{dt}$$

$$= \dot{\bar{x}}^T(t)\bar{P}_i(t)\bar{x}(t) + \bar{x}^T(t)\dot{\bar{P}}_i(t)\bar{x}(t) + \bar{x}^T\bar{P}_i(t)\dot{\bar{x}}(t)$$

$$= \bar{x}^T(t)[\bar{A}_i^T\bar{P}_i(t) + \bar{P}_i(t)\bar{A}_i - \dot{\bar{P}}_i(t)]\bar{x}(t)$$

$$-\bar{x}^T(t)\bar{P}_i(t)\bar{B}_i\Theta(\zeta) - \Theta(\zeta)\bar{B}_i^T\bar{P}_i(t)\bar{x}(t)$$

$$= -\bar{x}^T(t)[\bar{Q} - \bar{P}_i(t)\bar{B}_i\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)]\bar{x}(t)$$

$$-\zeta^T\bar{R}\Theta(\zeta)-\Theta(\zeta)^T\bar{R}\zeta$$

if $\zeta^T\cdot\bar{R}\cdot\Theta(\zeta)\geq l\cdot\zeta^T\cdot\bar{R}\cdot\zeta$

$$\dot{V}_i[\bar{x}(t)]\leq-\bar{x}^T(t)[\bar{Q}-\bar{P}_i(t)\bar{B}_i\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)]\bar{x}(t)$$

$$-2(\frac{1}{2}+\varepsilon(t))\zeta^T\bar{R}\zeta$$

$$=-\bar{x}^T(t)[\bar{Q}-\bar{P}_i(t)\bar{B}_i\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)]\bar{x}(t)$$

$$-\zeta^T\bar{R}\zeta-2\varepsilon(t)\zeta^T\bar{R}\zeta$$

$$=-\bar{x}^T(t)\bar{Q}\bar{x}(t)+\bar{x}^T(t)\bar{P}_i(t)\bar{B}_i\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)\bar{x}(t)$$

$$-[\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)\bar{x}(t)]^T\bar{R}[\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)\bar{x}(t)]$$

$$-2\varepsilon(t)\zeta^T\bar{R}\zeta$$

$$=-\bar{x}^T(t)\bar{Q}\bar{x}(t)+\bar{x}^T(t)\bar{P}_i(t)\bar{B}_i\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)\bar{x}(t)$$

$$-\bar{x}^T(t)\bar{P}_i(t)\bar{B}_i\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)\bar{x}(t)-2\varepsilon(t)\zeta^T\bar{R}\zeta$$

$$=-\bar{x}^T(t)\bar{Q}\bar{x}(t)$$

$$-2\varepsilon(t)[\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)\bar{x}^T]^T\bar{R}[\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)\bar{x}(t)]$$

$$=-\bar{x}^T(t)[\bar{Q}+2\varepsilon(t)\bar{P}_i(t)\bar{B}_i\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)]\bar{x}(t)$$

assume $\varepsilon(t)>0$

so

$$\bar{Q}+2\varepsilon(t)\bar{P}_i(t)\bar{B}_i\bar{R}^{-1}\bar{B}_i^T\bar{P}_i(t)>0$$

that means:

$$\dot{V}[\bar{x}(t)]<0$$

and also:

$$V_{i+1}[\bar{x}(t_i)]-V_i[\bar{x}(t_i)]$$

$$=\bar{x}^T(t)\bar{P}_{i+1}(t)\bar{x}(t)-\bar{x}^T(t)\bar{P}_i(t)\bar{x}(t)$$

$$=\bar{x}^T(t)[\bar{P}_{i+1}(t_i)-\bar{P}_i(t_i)]\bar{x}(t)=0, i=1,2,\cdots K+1$$

and $V_i(x)\to 0$ as $x\to\infty$

So the closed-loop linear switched system consisted of nonlinear uncertainty by the dynamic programming algorithm is robust stable in the sense of Lyapunov folllowing the theorem 5.1.

VI. EXAMPLE

In this section, a numerical example has been shown to illustrate the main algorithm we obtained in section 3.

Consider a switched linear system of the form:

$$subsystem1:\dot{x}(t)=\begin{pmatrix}1 & 2\\ -1 & 4\end{pmatrix}x(t)+\begin{pmatrix}1\\ 4\end{pmatrix}u(t)$$

$$y(t)=\begin{pmatrix}1 & 0\end{pmatrix}x(t) \tag{29}$$

Then, we assume that the linear time-invariant switched system as followed:

$$\dot{z}(t)=\begin{pmatrix}1 & 2.1\\ -2 & 4\end{pmatrix}z(t)$$

$$\tilde{y}(t)=\begin{pmatrix}1 & 1\end{pmatrix}z(t) \tag{30}$$

The output of linear time-invariant switched system above is $\tilde{y}$, which is tracked by y .

$$subsystem2:\dot{x}(t)=\begin{pmatrix}-1 & 2\\ 0.2 & 3\end{pmatrix}x(t)+\begin{pmatrix}2\\ -1\end{pmatrix}u(t)$$

$$y(t)=\begin{pmatrix}0 & 1\end{pmatrix}x(t) \tag{31}$$

Then, we assume that the linear time-invariant switched system as followed:

$$\dot{z}(t)=\begin{pmatrix}-1 & 2\\ 0.2 & 3\end{pmatrix}z(t)$$

$$\tilde{y}(t)=\begin{pmatrix}1 & 1\end{pmatrix}z(t) \tag{32}$$

The output of linear time-invariant switched system above is $\tilde{y}$, which is tracked by y ,and $x(t_f)=\begin{pmatrix}4 & 4\end{pmatrix}'$.

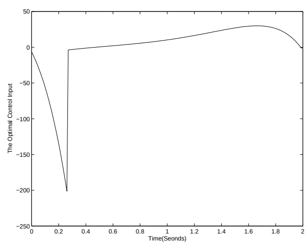
Assume that $t_0=0, t_f=2$, and the system switches once at $t=t_1(0\leq t_1\leq t_f)$ from subsystem 1 to 2 ,we want to find optimal t_1 and u such that:

$$J=\frac{1}{2}x^T(t_f)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}x(t_f)+$$

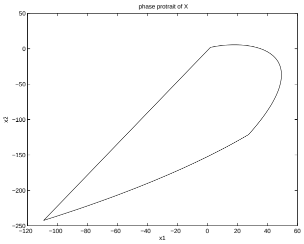
$$\frac{1}{2}\int_{t_0}^{t_f}\left[(y-\tilde{y})^T\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}\cdot(y-\tilde{y})+u^T(t)\cdot u(t)\right]dt \tag{33}$$

is minimal.

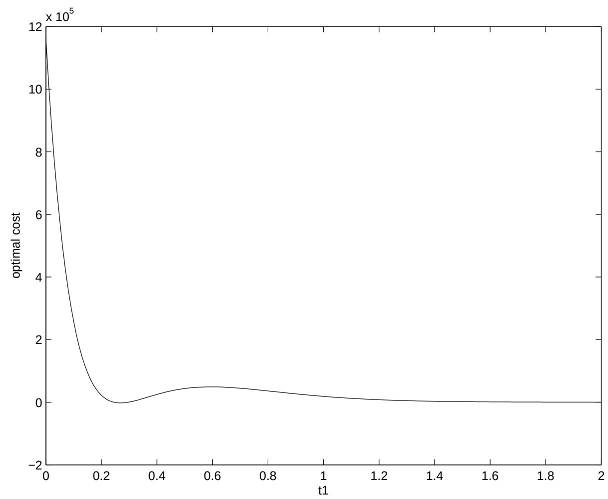
We use the main algorithm above to obtain the optimal switching instant t_1 from subsystem 1to subsystem 2 is 0.27s ,and the corresponding optimal cost is 0.000 . Fig.4 (b)shows the state trajectory of the switched tracking system. Fig.4 (a)shows the optimal control input. Fig.4 (c) shows the optimal cost for different t_1 's. Fig.4 (d)shows the tracking output y and $\tilde{y}$, therefore by utilizing a small initial undershoot the output rapidly tracks the reference, without large transient errors or undershoots.



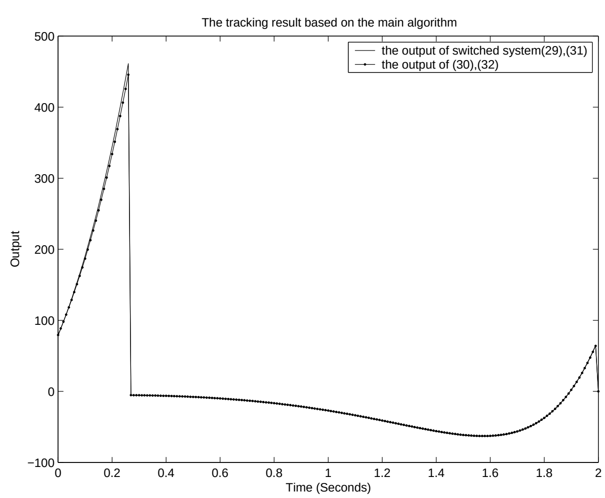
(a)



(b)



(c)



(d)

Fig. 4. (a) the optimal control input.(b) the state trajectory.(c)the optimal cost for different t_1 's.(d)the tracking output y and $\tilde{y}$

VII. CONCLUSIONS

In this paper, we studied optimal tracking problems for switched systems. Based on the dynamical programming, we proposed a method to obtain the tracking sensitivity function with important implications for the fidelity of reference tracking.Faced with this closed-loop system, we analysis it's stability in the sense of Lyapunov,then a sufficient condition is obtained to show the robust stability result. This method is particularly effective in the case of general switched linear quadratic problem.

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