Quasi-Min-Max MPC with linear Parameter-Dependent State Feedback Law

Zou Zhiqiang and Xu Lihong
Dept. of Control Science & Engineering, Tongji University
Shanghai 200092, the People's Republic of China
yuanmeng0305@163.com

Yuan Meng
Basic Dept., The Second Artillery Engineering Institute
Xi-an 710025, the People's Republic of China
myuan@iipc.zju.edu.cn

***Abstract*— Controlled invariance terminal region and associated feedback law are important in model predictive control of constrained systems, since large terminal region implies large region of stable initial conditions. In this paper, the polytopic invariance terminal region is designed instead of former ellipsoid terminal region, which enables greater flexibility in the shape of terminal region and could enlarge the invariance terminal region; at the same time the linear parameter-dependent state feedback law inside terminal region is designed instead of only the state feedback law, which enables higher degree of the feedback law and enlarges the invariance terminal region. In addition, one horizon free control is designed by utilizing real-time measured parameter-varying state space matrices, which enlarges the region of stable initial conditions and reduces the performance. Simulation expounds these characteristics**

***Index Terms*— MPC, Polytopic Terminal Region, Quasi-Min-Max, LP**

I. Introduction

Model Predictive Control(MPC) solves online a constraint optimization problem at each sampling instant and implements only the firstly element of the optimal control sequence. The optimization is repeated at the next sampling time by updating the initial condition with the new state ([1]). In the past most industrial applications have used the finite horizon implementation of MPC. However, despite many reported successful applications, the finite horizon MPC algorithms are difficult to analyze theoretically since closed-loop asymptotic stability depends on many tuning parameters in an unnecessarily complicated way and no guarantees are provided ([2], [3]).

Realizing this drawback, some researchers have revisited MPC and studied it as a constrained infinite horizon control problem, which led to useful stability results. To meet the demand of the real-time optimization, a switching dual-mode horizon controller is designed to guarantee the stability, in which a local stable feedback control law is applied once the states enter an invariance terminal region and a finite horizon predictive controller is applied outside the terminal region ([4]-[7]). The local stability is proven by the existence of the feasible terminal region.

In this paper, we consider the class of linear parameter-varying(LPV) systems whose state space matrices vary inside the prescribed polytope and are real-time measured. In recent years, there has been a renewed interest in LPV system, which mostly was researched based on switching dual-model MPC. The invariance terminal region was designed as ellipsoid set ([3], [5], [8]-[10]). Here we design polytopic invariance terminal region instead of the ellipsoid terminal region and the parameter-dependent feedback law correlative with the state and the parameter-varying state space matrices instead of only the state feedback law, which not only enables greater flexibility in the shape of terminal region and a higher degree of the feedback law, but also could significantly enlarge the invariance terminal region. In addition, one horizon free control is designed by utilizing real-time measured parameter-varying state space matrices, which enlarges the region of stable initial conditions and reduces the performance.

Section 2 defines the basic system of interest as a polytopic LPV model. Section 3 designs polytopic invariance terminal region and linear parameter-dependent state feedback law. Section 4 designs one horizon free control. In Section 5, simulations on a numerical example illustrate the application of the algorithm. Finally, Section 6 concludes the paper.

II. Problem Statement

The model considered here is the following LPV system with polytopic uncertainty:

$$x(k+1) = A(k)x(k) + Bu(k) \tag{2.1}$$

$$y(k+1) = C(k)x(k) \tag{2.2}$$

Where $x(k) \in R^{n_x}$ denoting the state of the plant, $u(k) \in R^{n_u}$ the control input, $y(k) \in R^{n_y}$ the output with constraints

$$\|c_m y(k) + d_m u(k)\|_\infty \leq 1, \quad m = 1, 2, \cdots, M \tag{2.3}$$

Here $\|c_m y(k) + d_m u(k)\|_\infty$ denotes the maximum absolute value of the element in the vector $c_m y(k) + d_m u(k)$.

In this paper, we assure that for any k, the parameter-varying state space matrices

$$[A(k), C(k)] \in \Sigma = Co\{[A_1, C_1], \cdots, [A_L, C_L]\} \tag{2.4}$$

Namely, existing nonnegative coefficients $\lambda_l(k)$ satisfying

$$\sum_{l=1}^{L} \lambda_l(k) = 1, \quad [A(k), C(k)] = \sum_{l=1}^{L} \lambda_l(k)[A_l, C_l] \tag{2.5}$$

Moreover, we also assume no mismatch exists between the model and the plant, at each sampling instant k the plant state and the state space matrices $[A(k),C(k)]$ are available in real-time and the future state space matrices $[A(k+t),C(k+t)](t \geq 1)$ are uncertain and vary inside the prescribed polytope Σ.

MPC consists of a step-by-step optimization technique. At each step new measurements are obtained and a cost function depending on the predicted future outputs of the plant is minimized. Let $x(k+t|k)$ be the predictive state of the plant at time $k+t$, $y(k+t|k)$, $u(k+t|k)$ the future output, control input at time $k+t$ respectively. We aim at designing controller steering the state into stable point and minimizing the following infinite horizon performance that is split into two parts.

$$J_k = \sum_{t=0}^{N-1} \|y(k+t|k)\|_\infty + \sum_{t=N}^{\infty} \|y(k+t|k)\|_\infty \tag{2.6}$$

III. Polytopic Invariant Set

For infinite horizon optimal control problem (2.6), to meet the demand of online optimization, we always adopt dual-mode horizon controller, Therefore, at every time k, a "Quasi-Min-Max" MPC minimizes the following worst-case performance subject to the LPV model (2.1) and (2.2).

$$\min_{U_0^\infty(k)} \max_{[A(k+t),C(k+t)]\in\Sigma,\, t\geq 0} J_k \tag{3.1}$$

The control inputs are denoted by $U_0^k = \{u(k|k), \cdots, u(k+N-1|k), U_N^\infty\}$, where $\{u(k|k), \cdots, u(k+N-1|k)\}$ is the finite horizon control sequence outside the terminal region, U_N^∞ is the rest of the future control in terminal region. In previous literature, the terminal region is commonly an ellipsoid set based on LQR problem by the linear state feedback law

$$U_N^\infty : \{u(k+t|k) = F(k)x(k+t|k),\, t \geq N\} \tag{3.2}$$

or by the following linear parameter-dependent state feedback law.

$$U_N^\infty : \{u(k+t|k) = \sum_{l=1}^{L} F_l(k)x(k+t|k),\, t \geq N\} \tag{3.3}$$

The state feedback law $F(k)$(or $F_l(k)$) and the ellipsoid set can be computed online ([7], [8]) based on semi-define program subject to linear matrix inequalities (LMIs). To meet the demand of the online optimization, we always off-line compute the ellipsoid set and feedback law ([6], [11]).

Controlled invariance terminal region and associated feedback law are important in MPC, since large terminal region implies large region of stable initial conditions. In this paper we design polytopic invariance terminal region instead of the ellipsoid region.

Let polytopic region $\Omega = \{x \in R^{n_x} | \|Vx\|_\infty \leq 1\}$, then Ω is positively invariant under (2.1) and (2.2) with a given feedback law $\kappa_l(x)$ at each vertex of the prescribed polytope Σ if the following conditions hold for any $x \in \Omega$,

$$\|V(A_l x + B\kappa_l(x))\|_\infty \leq 1,\ l = 1,2,\cdots,L \tag{3.4}$$

$$\|c_m C_l + d_m \kappa_l(x)\|_\infty \leq 1,\ m = 1,2,\cdots,M \tag{3.5}$$

Here invariance of Ω therefore follows from (3.4). More restrictive invariance conditions shows as the following Lemma.

Lemma 3.1 *If (3.5) holds, and for $l = 1,2,\cdots,L$,*

$$\|V(A_l x + B\kappa_l(x))\|_\infty + \frac{1}{\tau}\|C_l x\|_\infty \leq \|Vx\|_\infty,\ \forall x \in \Omega \tag{3.6}$$

is satisfied for some $\tau > 0$, then Ω is positively invariant, and for any $x(k|k) \in \Omega$, the system (2.1), (2.2) is stable and "Quasi-Min-Max" problem of (3.1) has upper bound

$$\min_{U_0^\infty(k)} \max_{[A(k+t),C(k+t)]\in\Sigma,\, t\geq 0} J_k \leq \tau\|Vx(k|k)\|_\infty \tag{3.7}$$

with linear parameter-dependent state feedback law

$$u(k+t|k) = \sum_{l=1}^{L} \lambda_l(k+t)\kappa_l(x(k+t|k)) \tag{3.8}$$

Where the nonnegative coefficients $\lambda_l(k+t)$ satisfy (2.5).

Proof: Invariance of Ω is ensured by the conditions (3.5) and (3.6). If $x(k|k) \in \Omega$, $x(k+t|k) \in \Omega(t \geq 0)$. With the parameter-dependent state feedback law of (3.8), we have

$$\|Vx(k+t+1|k)\|_\infty + \frac{1}{\tau}\|C(k+t|k)x(k+t|k)\|_\infty$$

$$\leq \sum_{l=1}^{L} \lambda_l(k+t)\|Vx(k+t|k)\|_\infty = \|Vx(k+t|k)\|_\infty$$

Then the system (2.1), (2.2) is stable and the upper bound of "Quasi-Min-Max" problem (3.1) is obtained by summing (3.6) over all times $t \geq 0$.

The polytopic invariance terminal region could significantly enlarge the invariance terminal region, but could increase the complexity of the volume maximization problem. To avoid this difficulty, we use the concept of partial invariance defined as follows.

Definition 3.2 *(Partial Invariance [12]) $\Omega^{(-1)} \subset R^{n_x}$ is partially invariant for $x(k+1) = f(x(k), \kappa(x(k)))$, if $f(x(k), \kappa(x(k))) \in \Omega^{(0)}\ \forall x(k) \in \Omega^{(-1)}$ where $\Omega^{(0)} \subset \Omega$ for some invariant set Ω.*

Conditions for partial invariance can be extended in an obvious way to ensure the bound on performance (See theorem 3.5). Let $\Omega^{(\alpha)} = \{x : \|V^{(\alpha)}x\|_\infty \leq 1\}(\alpha = 0, -1)$, then for $l = 1,2,\cdots,L$,

$$\|V^{(0)}(A_l x + B\kappa_l(x))\|_\infty + \frac{1}{\tau}\|C_l x\|_\infty \leq \|V^{(-1)}x\|_\infty \tag{3.9}$$

Where the invariance set $\Omega^{(0)}$ satisfies (3.5) and (3.6). $\forall x \in \Omega^{(-1)}$ ensures that $\Omega^{(-1)}$ is partial invariance. Because of the convex of $\Omega^{(-1)}$, we can give a convenient reformulation of (3.9).

Theorem 3.3 *The following conditions, invoked for $l = 1, 2, \cdots, L$, are equivalent to (3.9)*

$$\|V^{(0)}(A_l\beta_j^{(-1)} + Bu_{l,j}^{(-1)})\|_\infty + \frac{1}{\tau}\|C_l\beta_j^{(-1)}\|_\infty \le 1 \quad (3.10)$$

with linear state feedback law

$$\kappa_l(k) = \|V^{(-1)}x\|_\infty \sum_{j=1}^{p} q_j^{(-1)} u_{l,j}^{(-1)} \quad (3.11)$$

Where $\beta_j^{(-1)}(j = 1, 2, \cdots, p)$ are the vertices of $\Omega^{(-1)}$, the nonnegative coefficients $q_j^{(-1)}$ satisfy

$$\sum_{j=1}^{p} q_j^{(-1)} = 1,\ x = \|V^{(-1)}x\|_\infty \sum_{j=1}^{p} q_j^{(-1)}\beta_j^{(-1)}.$$

Proof: The necessity of (3.10) is obvious since (3.9) must hold for all $x \in \Omega^{(-1)}$. To show sufficiency, for any $x \in \Omega^{(-1)}$, if (3.10) holds, with feedback law of (3.11), we have

$$\|V^{(0)}(A_l x + B\kappa_l(x))\|_\infty + \frac{1}{\tau}\|C_l x\|_\infty \le \|V^{(-1)}x\|_\infty.$$

To determine the volume of a polytopic set Ω as a function of its vertices can be computationally demanding, and to avoid this, we propose volume maximization by successively optimizing individual vertex of Ω. Namely, consider the problem of optimizing a vertex $\beta_j^{(-1)}$ with remaining vertices $\{\beta_r^{(-1)},\ r \ne j\}$ fixed. Given an initial polytope $\hat{\Omega}$ with vertices $\{\hat{\beta}_j^{(-1)}, (\beta_r^{(-1)})_{r\ne j}\}$, this is equivalent to $\max_{\beta_j^{(-1)}} vol(\Omega - \hat{\Omega})$, which can be expressed as the sum of volumes of all simplexes formed by $\beta_j^{(-1)}$ with the faces of $\hat{\Omega}$ containing $\hat{\beta}_j^{(-1)}$. Denoting X_{jk} as the matrix with columns formed from the vertices of the kth facet, $\overline{1} = [1, \cdots, 1]^T$, it follows that ([12], [13])

$$\begin{aligned} vol(\Omega - \hat{\Omega}) &= \frac{1}{n}\sum_k det(X_{jk} - \beta_j^{(-1)}\overline{1}^T) \\ &= \frac{1}{n}\sum_k det(X_{jk}) - \overline{1}^T X_{jk}^{-T}\beta_j^{(-1)}) \end{aligned}$$

Since X_{jk} is independent of $\beta_j^{(-1)}$, maximization of $vol(\Omega - \hat{\Omega})$ is equivalent to minimizing the linear objective

$$\varpi_j^T\beta_j^{(-1)}, \quad \varpi = \sum_k X_{jk}^{-T}\overline{1}$$

Theorem 3.4 *The volume of $\Omega^{(-1)}$ is maximized over $\beta_j^{(-1)}, u_{i,j}^{(-1)}$ for fixed $\tau > 0$ by the following linear program (LP).*

$$\min_{\beta_j^{(-1)}, u_{l,j}^{(-1)}} \varpi_j^T\beta_j^{(-1)} \qquad s.t. \quad (3.12)$$

$$\begin{cases} \|c_m C_l\beta_j^{(-1)} + d_m u_{l,j}^{(-1)}\|_\infty \le 1 \\ \|V^{(0)}(A_l\beta_j^{(-1)} + Bu_{l,j}^{(-1)}\|_\infty + \frac{1}{\tau}\|C_l\beta_j^{(-1)}\|_\infty \le 1 \\ m = 1, 2, \cdots, M, \quad l = 1, 2, \cdots, L \end{cases}$$

Because of the symmetry of $\Omega^{(-1)}$, we here choose even number vertices $\{\beta_1^{(-1)}, \cdots, \beta_{p/2}^{(-1)}, -\beta_1^{(-1)}, \cdots, \beta_{p/2}^{(-1)}\}$ in $\Omega^{(-1)}$. The method of optimizing individual polytope vertex in theorem 3.4 suggests the following procedure for appropriately maximizing the volume of $\Omega^{(-1)}$.

Algorithm 3.5(Enlargement of the polytopic set)
DATA: $\Omega^{(0)} = \{\beta_j^{(0)},\ j = 1, 2, \cdots, p\}$ and $\kappa_l(x)$ satisfying (3.5), (3.6) for fixed $\tau \ge 0$.

1) *Determine the linear objective in (3.12);*
2) *Solve the LP (3.12) for $\{\beta_j^{(-1)}, u_{l,j}^{(-1)}\}$;*
3) *If $j < p/2$, $j = j+1$, return 1); else $\beta_j^{(-1)} = -\beta_{j-p/2}^{(-1)}$.*

A sequence of partially invariant sets $\Omega^{(-P)}, \cdots, \Omega^{(-1)}$ with vertices $\{\beta_j^{-i}\}_{j=1}^{p}(i = 1, \cdots, P)$ can be computed through successive application of Algorithm 3.5 based on initial polytopic sets $\Omega^{(-P+1)}, \cdots, \Omega^{(0)}$, and then we hold following theorem.

Theorem 3.6 *Let $\{\beta_j^{(-i)}\}_{j=1}^{p}$ be the vertices of $\Omega^{(-i)}(i = 0, 1, \cdots, P)$, then $\Omega = Co\{\beta_j^{(-i)}\}(j = 1, 2, \cdots, p,\ i = 0, 1, \cdots, P)$ is the positively invariance terminal region of system (2.1) and (2.2). For any $x(k|k) \in \Omega$, the system (2.1), (2.2) is stable with the linear parameter-varying state feedback law*

$$u(k+t|k) = \sum_{l=1}^{L} \lambda_l(k+t)\kappa_l(x(k+t|k)) \quad (3.13)$$

and "Quasi-Min-Max" problem of (3.1) has upper bound

$$\min_{U_0^\infty(k)} \max_{[A(k+t),\, C(k+t)] \in \Sigma,\, t \ge 0} J_k \le \tau\|Vx(k|k)\|_\infty \quad (3.14)$$

Where nonnegative coefficient $\lambda_l(k+t)$ satisfy (2.5) and the state feedback law

$$\kappa_l(x(k+t|k)) = \|Vx(k+t|k)\|_\infty \sum_{j=1}^{p}\sum_{i=0}^{P} q_j^{(-i)} u_{l,j}^{(-i)} \quad (3.15)$$

with nonnegative coefficients $q_j^{(-i)}$ satisfying

$$\sum_{j=1}^{p}\sum_{i=0}^{P} q_j^{(-i)} = 1,\ \ x = \|Vx\|_\infty \sum_{j=1}^{p}\sum_{i=0}^{P} q_j^{(-i)}$$

Proof: For any $x \in \Omega$, with state feedback law of (3.15) and $A_l\beta_l^{(-i)} + Bu_{l,j}^{(-i)} \in \Omega$, we have

$$\|V(A_l x + B\kappa_l(x)\|_\infty + \frac{1}{\tau}\|C_l x\|_\infty \le \|Vx\|_\infty$$

With lemma 3.1, we know $\Omega = Co\{\beta_j^{(-i)}\}(j = 1, 2, \cdots, p,\ i = 0, 1, \cdots, P)$ is the invariance terminal region and (3.14) holds.

Remark 3.7 To meet the demand of the online optimization, the polytopic invariance terminal region and the linear parameter-dependent feedback law are computed offline. In addition, the invariance terminal region Ω closely depends on the parameter τ, large τ could increase the volume of invariance terminal region and the "Quasi-Min-Max" performance of (3.1). To sort with the performance and the feasibility, we offline compute invariance terminal region $\Omega_s = \{x \mid \|V_s x\|_\infty \leq 1\}$ and associated linear parameter-dependent state feedback law based on different coefficient $\tau_s (s = 1, 2, \cdots, S)$.

IV. One Horizon Free Control

Given a sequence of invariance terminal region $\Omega_s (s = 1, \cdots, S)$ computed off-line using the Algorithm 3.5, at each sampling instant k, one horizon free control is designed by utilizing real-time measured state space matrices $[A(k), C(k)]$. To ensure the stability of (2.1) and (2.2), we hope

$$\|V_s x(k+1|k)\|_\infty \leq 1 \tag{4.1}$$

Then the following control strategy determines the region Ω_s containing $x(k+1|k)$ with the smallest associated cost bound and then implement the one horizon free control.

Algorithm 4.1 (One horizon free control)

1) *Determine* $u = arg_u \, min_s \; \tau_s \|V_s(A(k)x(k) + Bu)\|_\infty$, *where arg means the control which leads to smallest cost of* $\tau_s \|V_s(A(k)x(k) + Bu)\|_\infty$;
2) *Implement* $u(k|k) = u$.

The invariance of Ω_s will ensure the stability properties, then we get the following theorem

Theorem 4.2 *If the Algorithm 4.1 is feasible for some* $x(0)$ *at* $k = 0$, *then Algorithm 4.1 ensures that the closed-loop cost (2.6) is bounded by the optimal value of the objective in (3.1) at* $k = 0$, *and renders the equilibrium at* $x = 0$ *asymptotically stable.*

Proof: Let the optimal value of the "Quasi-Min-max" performance, corresponding one horizon free control and polytopic terminal region at time k be $J_\infty(k)$, $u(k|k)$ and Ω_s. A feasible but suboptimal input sequence at $k+1$ is given by $\{\kappa_s(x(k+t|k), t \geq 1\}$, where $\kappa_s(x)$ is the linear parameter-dependent state feedback law of (3.15) in Ω_s, then the initial feasibility ensures future feasibility. Let the "Quasi-Min-Max" cost be J'_{k+1} corresponding to this suboptimal predictive input sequence and the optimal value of the "Quasi-Min-max" objective be J_{k+1}, then the following condition holds

$$J_{k+1} \leq J'_{k+1} \leq J_k - \|y(k)\|_\infty$$

Which implies that the closed-loop cost is bound by J_0 , and that $J_k(x(k|k))$is a Lyapunov function demonstrating stability of $x = 0$. Furthermore the bound $\sum_{k=0}^{\infty} \|y(k|k)\|_\infty \leq J_0$ implies asymptotic convergence $\|y(k|k)\|_\infty \to 0$ and hence $x(k|k) \to 0$.

Remark 4.3 Compared with the fixed linear parameter-dependent state feedback law, MPC with one horizon free control has higher degree of freedom, which enlarges the region of stable initial conditions and reduces the "Quasi-Min-Max" performance of (3.1). In addition, we can choose larger horizon free control, i.e. setting $N > 1$ in (2.6)(see the detail in [7]).

V. Simulation

Example 5.1 Considering discrete LPV system in R^2

$$x(k+1) = \begin{bmatrix} x_1(k)/2 & 0 \\ 1/2 + sin^2(k)/2 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$y(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(k)$$

With the control constraint $\|u(k)\|_\infty \leq 1$, the input constraint $\|x(k)\|_\infty \leq 2$, It's easy to know that the parameter-varying state space matrices varying inside the prescribed polytope $\Sigma = \left\{ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1/2 & 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 1 & 0 \\ 1/2 & 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \right\}$. Then the region of stable initial conditions $\Pi = \{0\}$ if adopting MPC with fixed state feedback law, and the associated regions of stable initial conditions adopting other MPC are shown in Fig.1.

VI. Conclusion

In this paper we propose a novel "Quasi-Min-Max" MPC based on polytopic invariance terminal region. Compared with the former methods, our MPC has following characteristics.

1) Designing polytopic invariance terminal region instead of ellipsoid terminal region , which enables

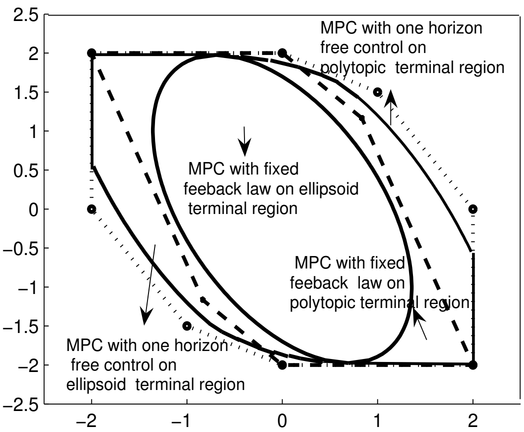

Fig. 1. Regions of stable initial conditions adopting different MPC (fixed feedback law, one horizon free control) with different terminal regions (ellipsoid, polytope with six vertices)

greater flexibility in the shape of terminal region and could enlarge the invariance terminal region;

2) Designing the linear parameter-dependent state feedback law inside terminal region instead of only state feedback law, which enables higher degree of the feedback law and enlarges invariance terminal region;
3) Designing one horizon free control, which enlarges the region of stable initial conditions and reduces the plant performance.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the contribution of Ph. D sustentation fund of the Ministry of Education, the People's Republic of China (No: 20020247023).

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