Stochastic/Adaptive Sliding Mode Observer for Noisy Excessive Uncertainties Nonlinear Systems

Reza Raoufi
Department of Electrical Engineering,
Ferdowsi University of Mashhad, Mashhad, Iran
rezaraoufi@cic.aut.ac.ir

Hamid Khaloozadeh
Department of Electrical Engineering,
K.N.Toosi University of Technology, Tehran, Iran
h_khaloozadeh@kntu.ac.ir

Abstract: **A robust/adaptive stochastic observer is presented for stochastic nonlinear dynamics having excessive uncertainties. It was shown through a new theorem that the proposed nonlinear robust sliding mode observer has very accurate state estimate error characteristic. The observer uses the sliding mode technique for the robustness and a deterministic adaptive law to guarantees a globally asymptotically convergence observation error. Finally, an example is given to illustrate the application and very favorable convergence properties of the proposed observer.**

Index Tterms- Adaptive system, Observers, Stochastic theory, Sliding mode observer

1. INTRODUCTION

Observing the state hence the name "observer", is an important problem in the theory of systems. For linear systems, it has been extensively improved, and has proven extremely useful, especially for control applications such as observer-based-control design. For nonlinear systems, the theory of observers is not nearly as complete as it is for linear systems. The use of variable structure techniques in state reconstruction of nonlinear systems based on feedback linearization and extended linearization have been presented (Bestel and Zetiz, 1983; Krener, 1985; Isidori, 1985; Baumann and Rugh, 1986).A comparison of some of these techniques came to the conclusion that variable structure observers exhibit the best performance in this particular case study (Walcott and et al., 1987).

Therefore, robust techniques of state observation like sliding-mode observers have received ever increasing attention for linear and nonlinear systems (Koshkouei and Zinober, 1995; Sira and Spurgeon, 1994; Sira and et al., 1994; Utkin, 1992). The major feature of Robust sliding mode observer is the capability of reconstruction the states of a linear or nonlinear variable structure system with uncertainty having unknown bounded disturbances and measurement uncertainties (Slotine and Hedrick, 1986; Yaz and Azemi, 1994; Barbot and et al., 1996).It should be pointed out all the designed observers introduced above requires the knowledge of a bounding function on the uncertainties which should not be intensive and too excessive. A method for designing an adaptive observer for nonlinear deterministic systems has been presented (Yaz and Azemi, 1993) and other adaptive approaches in the context of variable-structure have been considered (Corless and Leitmann, 1983; Chen, 1989; Yoo and Chung, 1992).

In this work, nonlinear systems with excessive and strong uncertainties and unmodeled dynamics in presence of noise are considered. Designing a robust stochastic observer based on a new adaptive update law, Ito calculus and stochastic lyapunov stability (Florchinger, 1995; Gard, 1988; Mao1994) have been proposed. Finally, a simulation example will illustrate the efficacy and much more accurate observation of the new propose filter in comparison with the standard nonlinear estimation method of extended Kalman filter (EKF).

The following notation will be used in this paper. $x \in \boldsymbol{R^n}$ denotes an *n-vector* with real elements and associated norm, $\|x\| = (x^T x)^{1/2}$. $\boldsymbol{R^+}$ is the set of nonnegative real numbers. $\lambda_{\min}(A)\,(\lambda_{\max}(A))$ denotes the minimum (maximum) eigenvalue of asymmetric matrix $A(A \in R^{n\times n})$.the symbol **exp** or **e** is used for the exponential function and $E\{.\}$ denotes the expected value.

II. STOCHASTIC UNCERTAIN NONLINEAR SYSTEM MODEL

Consider the following nonlinear, nonautonomous system given by the Ito differential equation

$$dx_t = (Ax_t + Bu_t)dt + f(x_t,\zeta_t)dt + g_1(x_t)dv_t \qquad (1)$$

and the measurement equation

$$dy_t = Cx_t dt + g_2(x_t)dw_t\,,\; y_t \in \boldsymbol{R^m} \qquad (2)$$

where $t \in \boldsymbol{R^+}$, $x_t \in \boldsymbol{R^n}$ and $u_t \in \boldsymbol{R^p}$ is the known input signal, $\zeta_t \in \boldsymbol{R^n}$ the deterministic process disturbance that cannot be measured, $f, g_1, g_2 : \boldsymbol{R^n} \times \boldsymbol{R^+} \to \boldsymbol{R^n}$ that f represents the nonlinear and uncertain dynamics ,and also noise intensities in the state space model which are assumed to satisfy conditions like Lipschitz continuity and growth (Gard, 1988; Mao1994) are shown by g_1, g_2 . Lipschitz and growth conditions will guarantee existence and uniqueness of solutions.

It should be pointed out that g_1, g_2 are bounded as

$$\|g_1(x_t)\|_F \le \beta_1 , \|g_2(x_t)\|_F \le \beta_2 \tag{3}$$

v_t, w_t are the standard Wiener process noise independent of x_0 .

III. STOCHASTIC ROBUST ADAPTIVE OBSERVER

III.1. Model assumptions

If the following assumptions are considered, then the existence of a convergent observer will be guaranteed.

1) The pair (A,C) in (1) and (2) is detectable and observable. So an observer gain K exists such that $A-KC$ is a strictly Hurwitz matrix.

2) f is separable, i.e. $f = f_1 + f_2$ and f_1 satisfies a Lipschitz-like condition as follows:

$$\| f_1(x_1) - f_1(x_2)\| \le \ell \|x_1 - x_2\| \, ; \forall x_1, x_2 \in \boldsymbol{R^n} \tag{4}$$

where $\ell \in \boldsymbol{R}^+$ is a known constant, which must be determined.

3) f_2 is an excessive unknown bounded uncertainty or unmeasurable deterministic disturbance which can be modeled as

$$f_2(t) = C^T \zeta(t, x_t, u_t) \le \overline{\zeta} \tag{5}$$

III. 2. Lyapunov-based stochastic stability

Definition1: (Florchaingerhe, 1995)

The solution x_t of equation (1) is said to be stable in probability for $t \ge 0$ if for any $t_0 \ge 0, \varepsilon > 0$

$$\underset{x \to 0}{Lim} P \left\{ \sup \|x_{t,t_0}\| \right\} > \varepsilon\} = 0 \tag{6}$$

Here x_{t,t_0} denotes the sample path of the solution of equation (1) starting from a point x at time t_0 .So, intuitively, the definition implies that for a stable stochastic system, the probability to escape from a spherical region around the origin should be small for a small deviation from equilibrium state.

Definition2: (Florchaingerhe, 1995; Gard, 1988)

Consider the nonlinear/stochastic equation (1).Let $U \subset \boldsymbol{R^n}$ be a domain contains the origin, and assume that there exists a positive definite Lyapunov function $V : U \to \boldsymbol{R^+}$, twice continuously differentiable everywhere except possibly at the origin. Using derivative of V via Ito calculus along the trajectory of the system (1) based on infinitesimal generator:

$$LV(x,t) = ((Ax_t + Bu_t) + f(x_t, \zeta_t))dt \frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} + \frac{1}{2} trace(g_1 g_1{}^T \frac{\partial^2 V}{\partial x^2}) \tag{7}$$

if V satisfies for all $x \in U - \{0\}$: $\quad LV \le 0$ (8)

Then the trivial solution of equation (1) is stable in probability.

III. 3. Proposed observer

Theorem: The following nonlinear sliding mode adaptive/stochastic observer

$$d\hat{x}_t = (A\hat{x}_t + Bu_t + f_1(\hat{x}_t))dt + K(Ce_t)dt + S(\hat{x}_t, y_t, \phi_t, \hat{\eta}_i)dt \tag{9}$$

reconstructs the states of model (1) from measurement (2), via the nonlinear adaptive sliding mode gain:

$$S = p^{-1} C^T \left(\sum_{i=1}^{N} \hat{\eta}_i \Gamma_i(e) \right) \frac{\gamma C e_t}{\left\| \|\mathrm{Ce}_t\| - \phi_t \right\|} \tag{10}$$

The adaptation algorithm is based on the expected value of estimation error as:

$$d\hat{\eta}_i = \gamma_0 E\{\|Ce_e\|\}^i dt \tag{11}$$

The proposed observer (9) provides an estimation error, such that the sample path and the mean square error of the estimation, both are exponentially bounded as:

$$\underset{t \to \infty}{Lim} \operatorname{Sup} \left\{ \|\mathrm{e}_t\|^2 \right\} \le \lambda_{\min}^{-1}(P).B_1.\left(\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} - \ell \right) \tag{12}$$

$$if \quad \ell \lambda_{\max}(P) < \lambda_{\min}(Q) \tag{13}$$

where $\phi : \boldsymbol{R^+} \to \boldsymbol{R^+}$ which satisfies $\phi_t = \gamma \exp(-\beta t)$

$\gamma, \beta > 0$, $\Gamma_i(e_t) = \dfrac{E\{\|Ce_t\|\}^i}{\|Ce_t\|}$, ($\Gamma_i : \boldsymbol{R^+} \to \boldsymbol{R^+}$), observer error is $e_t = x_t - \hat{x}_t$, $B_1 = \beta_1{}^2 \lambda_{\max}(P) + \beta_2{}^2 \lambda_{\max}(K^T PK)$

and if $A-KC$ is stable, then for any positive definite $(Q=Q^T) \in \boldsymbol{R}^{n\times n}$ there exists a unique positive definite $(P=P^T) \in \boldsymbol{R}^{n\times n}$ such that

$$(A-KC)^T P + P(A-KC) = -2Q \qquad (14)$$

f_2 satisfies the following classical matching condition

$$f_2(x,t) = P^{-1}C^T \psi(y,\hat{x},t) \qquad (15)$$

where $\psi: \ \boldsymbol{R}^p \times \boldsymbol{R}^+ \to \boldsymbol{R}^p$, satisfies:

$$\left\|\psi(y,\hat{x},t)\right\| \le \gamma \sum_{i=1}^{N} \eta_i \Gamma_i(e_t) \le \overline{\Gamma} \qquad (16)$$

<u>Proof:</u> To investigate the convergence property of the proposed observer, consider the following positive definite Lyapunov function candidate

$$V(e_t,\hat{\eta}_i) = V_1(e_t) + V_2(\hat{\eta}_i) = e^T Pe + \frac{\gamma}{\gamma_0}\sum_{i=1}^{N}\left(\hat{\eta}_i(t)-\eta_i\right)^2 \qquad (17)$$

and according to the definition of the observation error and equations (1), (9), it can be written as

$$de_t = (A-KC)e_t dt + [f_1(x_t) - f_1(\hat{x}_t)]dt + [f_2 - S]dt + g_1 dv_t - Kg_2 dw_t \qquad (18)$$

To analyze the behavior of this stochastic differential equation, infinitesimal generator, equation (7), is considered as follows

$$LV(e_t,\hat{\eta}_{t_i}) = 2[(A-KC)e_t + (f_2 - S) + f_1(x_t) - f_1(\hat{x}_t)]^T Pe_t + trace(GG^T P) + 2\frac{\gamma}{\gamma_0}\sum_{i=1}^{N}\left(\hat{\eta}_i(t)-\eta_i\right).\dot{\hat{\eta}}_i(t)$$

where $G = [g_1 \quad -Kg_2]$

Using (4), (11), and (15) yields

$$\begin{aligned} LV(e_t,\hat{\eta}_{t_i}) &\le -2e_t^T Qe_t + 2[P^{-1}C^T\psi(y_t,\hat{x}_t,t) - S]^T Pe_t + \ell e_t^T Pe_t + trace[G^T PG] + 2\gamma\sum_{i=1}^{N}\left(\hat{\eta}_i(t)-\eta_i\right).E\left\{\left\|Ce_t\right\|\right\}^i \\ &\le -2\left\|e_t\right\|^2\lambda_{\min}(Q) + 2\ell\left\|e_t\right\|^2\lambda_{\max}(P) + 2\left\|Ce_t\right\|\left\|\psi\right\| - 2S^T Pe_t + trace[G^T PG] + 2\gamma\sum_{i=1}^{N}\left(\hat{\eta}_i(t)-\eta_i\right).E\left\{\left\|Ce_t\right\|\right\}^i \\ &\le -2\left\|e_t\right\|^2(\lambda_{\min}(Q) - \lambda_{\max}(P)) + 2\gamma\left\|Ce_t\right\|\gamma\sum_{i=1}^{N}\eta_i\frac{E\left\{\left\|Ce_t\right\|\right\}^i}{\left\|Ce_t\right\|} - 2S^T Pe_t + trace[G^T PG] + 2\gamma\sum_{i=1}^{N}\left(\hat{\eta}_i(t)-\eta_i\right).E\left\{\left\|Ce_t\right\|\right\}^i \end{aligned}$$

It should be pointed out the above result is derived using (16) and the fact that for a positive definite matrix such as M, the following relationship is resulted $\lambda_{\min}(M)\left\|z\right\|^2 \le z^T Mz \le \lambda_{\max}(M)\left\|z\right\|^2$. Therefore

$$LV \le -2\left\|e_t\right\|^2(\lambda_{\min}(Q) - \lambda_{\max}(P)) - 2S^T Pe_t + trace[G^T PG] + 2\gamma\sum_{i=1}^{N}\hat{\eta}_i(t).E\left\{\left\|Ce_t\right\|\right\}^i$$

Also using (3) and the properties of noise, and proposed sliding-mode surface (10), the following result is obtained for above inequality

$$\begin{aligned} LV \le &-\left\|e_t\right\|^2(\lambda_{\min}(Q) - \ell\lambda_{\max}(P)) + \beta_1^2\lambda_{\max}(P) + \beta_2^2\lambda_{\max}(K^T PK) + 2\gamma\sum_{i=1}^{N}\hat{\eta}_i(t).E\left\{\left\|Ce_t\right\|\right\}^i - 2\left(P^{-1}C^T\left(\sum_{i=1}^{N}\hat{\eta}_i\Gamma_i(e)\right)\frac{\gamma Ce_t}{\left\|\left\|\mathrm{Ce_t}\right\| - \phi_t\right\|}\right)^T .P.e_t \\ \le &-\left\|e_t\right\|^2(\lambda_{\min}(Q) - \ell\lambda_{\max}(P)) + \beta_1^2\lambda_{\max}(P) + \beta_2^2\lambda_{\max}(K^T PK) + 2\gamma\left(\sum_{i=1}^{N}\hat{\eta}_i(t).E\left\{\left\|Ce_t\right\|\right\}^i\right).\left(1 - \frac{\left\|Ce_t\right\|}{\left\|\left\|\mathrm{Ce_t}\right\| - \phi_t\right\|}\right) \end{aligned} \qquad (19)$$

It is obvious that the forth part of the above inequality is negative, because in the case

$$\begin{aligned} &\left\|Ce_t\right\| > \phi_t \\ &\lim_{t\to\infty}\left\|Ce_t\right\| > \lim_{t\to\infty}\phi_t = 0 \\ &\left(1 - \frac{\left\|Ce_t\right\|}{\left\|\left\|\mathrm{Ce_t}\right\| - \phi_t\right\|}\right) < 0 \Rightarrow \phi_t > 0 \end{aligned} \qquad (20)$$

which, is according to the definition of the function ϕ_t, it is always true, in the other case i.e., $\left\|Ce_t\right\| < \phi_t$ this condition leads to

$$\left\|Ce_t\right\| > 0.5\phi_t$$

thus

$$0.5\phi_t < \left\|Ce_t\right\| < \phi_t \qquad (21)$$

For the other terms of inequality (20), using the definition (17) and considering $LV_2 \le 0$ thus

$$LV \le LV_1$$

and

$$\begin{aligned} LV_1 &= -\left\|e_t\right\|^2(\lambda_{\min}(Q) - \ell\lambda_{\max}(P)) + B_1 \\ &\le -(\lambda_{\min}(Q) - \ell\lambda_{\max}(P))\frac{V_1}{\lambda_{\max}(P)} + B_1 \end{aligned}$$

therefore

$$E\{V(e_t)\} \le V(e_0) - \frac{B_1}{B_2})e^{-B_2 t} + \frac{B_1}{B_2}, t \ge 0$$

where $B_2 = \dfrac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} - \ell$

therefore, if $B_2 > 0$, by using the definition of $V_1(e_t)$ and omitting the transient response, it is obtained for steady state error that

$$\lim_{t\to\infty}\sup E\left\{\|e_t\|^2\right\} \le \lambda_{\min}{}^{-1}(P).B_1\left(\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} - \ell\right)$$

This means that estimation has a mean-square exponentially ultimately bounded estimation error.

IV. SIMULATION RESULTS

Example:

Consider the Lorenz attractor as a nonlinear system in the presence of excessive uncertainties and noises which is described as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-\sigma & \sigma & 0\\ r & -1 & 0\\ 0 & 0 & -b\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ -x_1x_3\\ +x_1x_2\end{bmatrix} + \begin{bmatrix}10\cos(6\pi t)\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}1\\ 1\\ 1\end{bmatrix}v(t)$$

$$y(t) = Cx(t) + w(t)\ ,\ C = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

where the strong bounded/unmodeled dynamics are given by term $\Delta f(t)^T = [10\cos(6\pi t)\quad 0\quad 0]$, which is in accommodation with (5). $v(t), w(t)$ are Gaussian white noise with variances equal to 0.002, 0.01 respectively and the pair (A, C) is completely observable. Consider the function f_1 as $f_1(x_t) = \begin{bmatrix}0 & -x_1x_3 & x_1x_2\end{bmatrix}^T$. Since f_1 must be satisfies a Lipschitz-like condition, it is necessary to calculate a Lipschitz constant over the convex set

$$W = \{x \in \boldsymbol{R}^3 \mid |x_1| < \delta_1,\ |x_2| < \delta_2,\ |x_3| < \delta_3\}$$

The Jacobian matrix is given by

$$\left[\frac{\partial f_1(x)}{\partial x}\right] = \begin{bmatrix}0 & 0 & 0\\ -x_3 & 0 & -x_1\\ x_2 & x_1 & 0\end{bmatrix}$$

Using $\|.\|_\infty$ for vectors in $\boldsymbol{R}^3$ and the induced matrix norm for matrices, so

$$\left\|\frac{\partial f_1}{\partial x}\right\|_\infty = \max\{0, |-x_1| + |-x_3|, |x_1| + |x_2|\}$$

all points in W satisfy

$$|x_1| + |x_3| \le \delta_1 + \delta_3 = \delta_{13}$$
$$|x_1| + |x_2| \le \delta_1 + \delta_2 = \delta_{12}$$

Hence

$$\left\|\frac{\partial f_1}{\partial x}\right\|_\infty \le \max\{\delta_{12}, \delta_{13}\}$$

A Lipschitz constant can be taken as $\ell = \max\{\delta_{12}, \delta_{13}\}$. Thus the nonlinear term of chaos model $f_1(x)$ satisfies Lipschitz-condition (3). Let consider the nonlinear system with the parameter set

$$\sigma = 10 \quad b = 1.25 \quad r = 28$$

and initial conditions set

$$x_1(0) = 1.3 \quad x_2(0) = -1.34 \quad x_3(0) = \text{-}0.087$$
$$\hat{x}_1(0) = -1 \quad \hat{x}_2(0) = -1 \quad \hat{x}_3(0) = -1$$

Following the procedure described in section III.3 with observer gain and Q-matrix are selected as follow respectively

$$K = \begin{bmatrix}10.71 & 100 & .2\end{bmatrix} \quad , \quad Q = \begin{bmatrix}5 & 5 & 3\\ 5 & 6 & 2\\ 3 & 2 & 5\end{bmatrix}$$

thus

$$P = \begin{bmatrix}0.2741 & 0.0678 & 0.0692\\ 0.0678 & 1.1210 & -0.4426\\ 0.0692 & -0.4426 & 3.9889\end{bmatrix} = P^T > 0\ ,$$

where above designing satisfies (13).Other designable parameters of adaptive term and sliding surface are as follows

$$\gamma = 0.1 \quad \gamma_0 = 0.001 \quad \beta = 0.001 \quad N = 3$$

Fig. 1 represents the actual states of nonlinear system and their estimates. Fig. 2 displays error in state estimation of nonlinear uncertain stochastic model using the proposed sliding mode observer.

The results of error in estimation via the proposed filter have been compared with those obtained from the extended Kalman filter in Fig. 2. It is obvious that the state estimation based on the proposed sliding mode observer is more accurate than the extended Kalman filter.

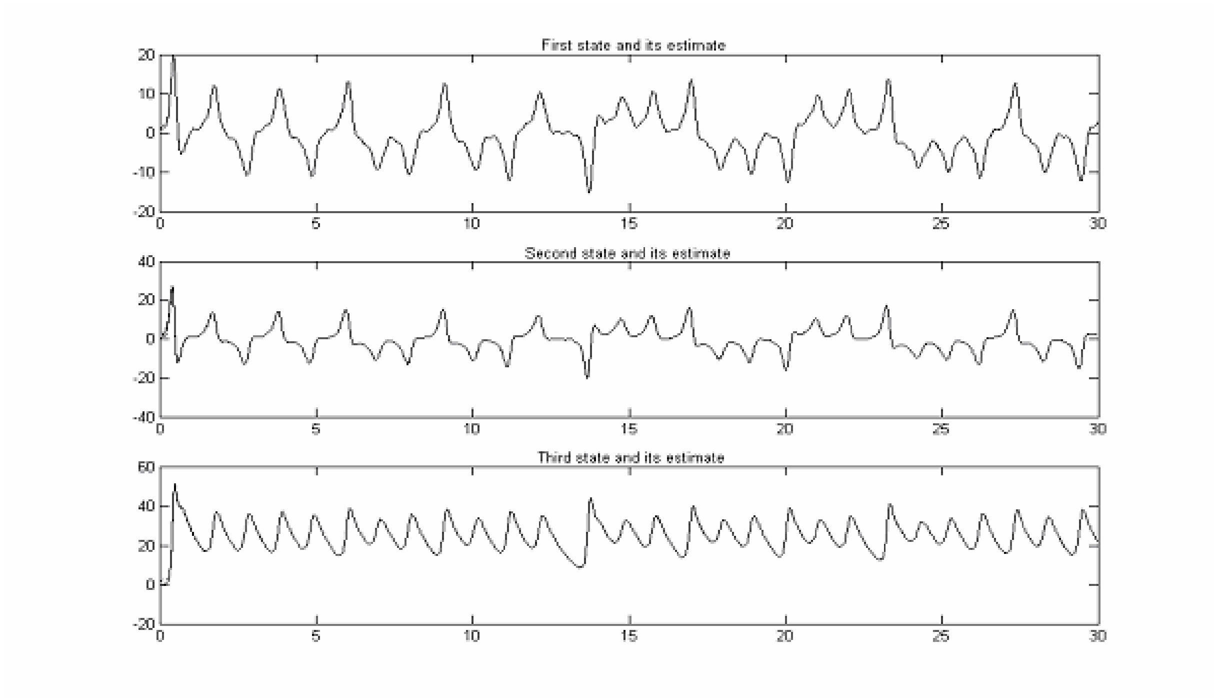

Fig. 1. Actual state (solid curve) of nonlinear uncertain system and their estimates (dashed curve) via proposed observer

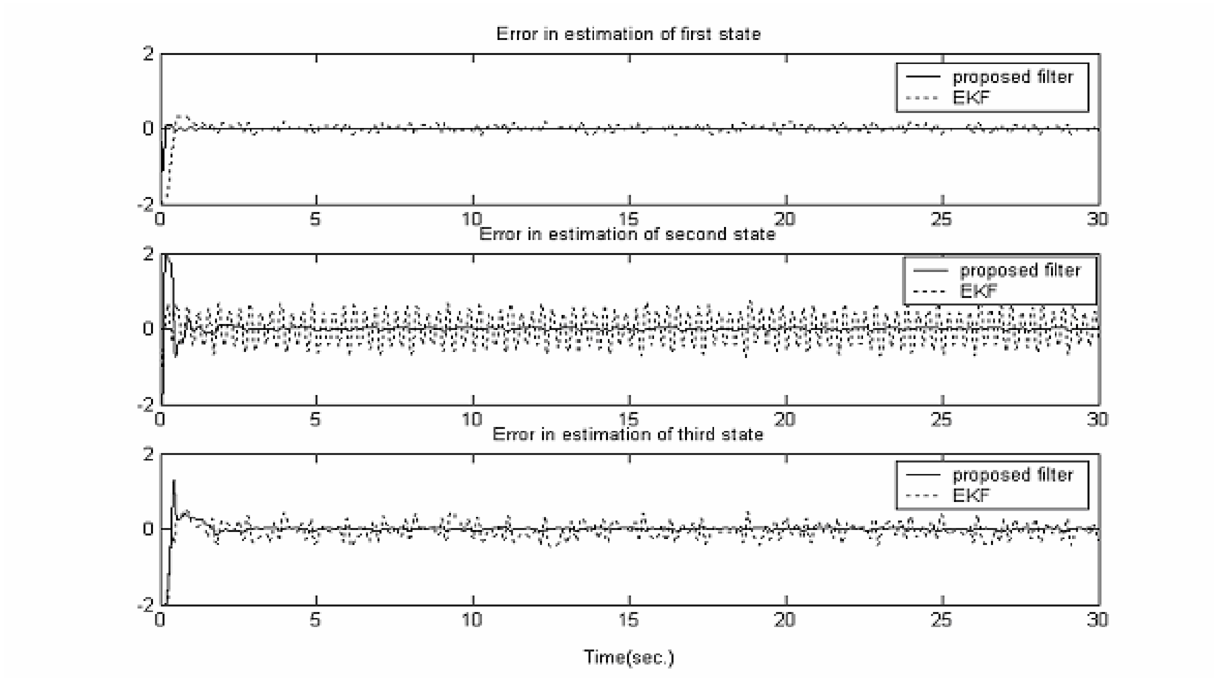

Fig. 2. State estimation based on proposed observer (solid curve) and state estimates via extended Kalman filter (dashed curve) of nonlinear uncertain system

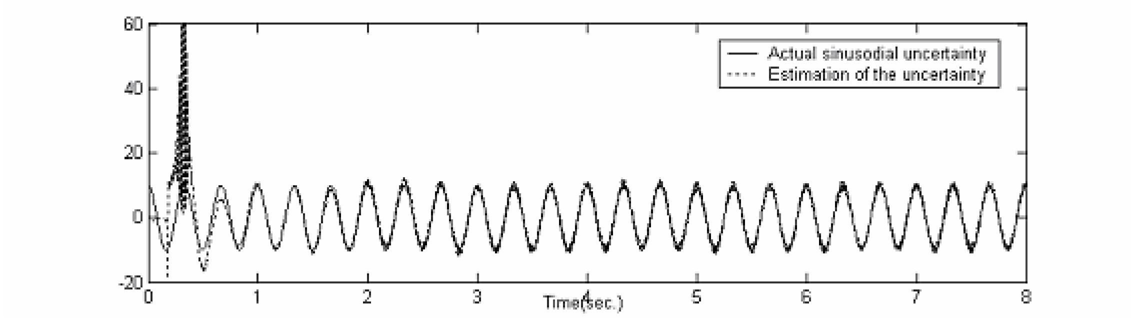

Fig. 3. Actual un-measurable uncertainty and its estimation based on adaptive sliding mode gain of the proposed observer

According to equations (1) and (9) the most accurate state estimation of stochastic uncertain model will be happened when

$$CS \approx f_2(t, x_t, \xi_t)$$

which mean that the proposed observer has the ability to estimate the unmodeling/uncertainties of a nonlinear model. This unique and very attractive feature of the proposed observer has been illustrated in Fig. 3.

V. CONCLUSIONS

A new nonlinear robust adaptive/stochastic sliding mode observer was presented which reconstructs the state of a nonlinear dynamic system with excessive bounded uncertainties and measurement noise. It was shown through a new theorem that the proposed nonlinear robust sliding mode observer has very accurate state estimate error characteristic. An example was given to illustrate the very satisfactory performance of this observer relative to one of the most commonly used filter (EKF) in this field.

REFERENCES

[1] Barbot J. P., T. Boukhobza, M. Djemai (1996). Triangular input observer form and sliding mode observer, IEEE CDC, pp 1489-1491.

[2] Baumann W., W. Rugh (1986). Feedback control of nonlinear systems by extended linearization. *IEEE Trans. Autom. Contr.,* Vol. **31**, pp.40-47.

[3] Bestle D., M. Zeitz (1983).Canonical form observer design for nonlinear time-variable systems. *Int. J Control.* Vol. **38**, pp. 419-431.

[4] Chen Y. H. (1989). Modified adaptive robust control system design. *Int. J. Control*, Vol. **49**, pp. 1869-1882.

[5] Corless M., G. Letimann (1983). Adaptive control of systems containing uncertain functions and unknown functions with uncertain bounds. *JOTA*, Vol**. 41**, pp. 155-168.

[6] Gard T. C. (1988). *Introduction to Stochastic Differential Equations*, Marcel Dekker: New York.

[7] Isidori A. (1985). *Nonlinear control Systems: An Introduction*, ,Springer-Verlag, Berlin.

[8] Koshkouei A. J. ,A. S. I. Zinober (1995). Sliding mode Observers for Linear Multivariable systems. *Proceeding of the 34th conference on decision & control.* New Orleans, LA, pp.2115-2120.

[9] Kerner A. J. ,W. Respondek (1985). Nonlinear observer with linearizable error dynamics. *J. Control & Optim.*, Vol. **23**,pp. 197-216.

[10] Lorchinger P. F. (1995). Luapunov techniques for stochastic stabiliti. *SIAM J. Contrl and Optim,* Vol.**33**, pp. 1151-1169.

[11] Mao X. (1994). *Stochastic Stability, Attraction and Boundedness.*

[12] Utkin V. I. (1992). *Sliding Mode in Control and Optimization.* Speringer-Verlag, Berlin.

[13] Walcott B. L., S. H. Zak (1987). State observation of uncertain nonlinear dynamical systems. *IEEE Trans. Automat. Control,* Vol.**32** pp, 166-170.

[14] Yaz E. E. ,A. Azemi (1994).Robust/Adaptive Observer for Systems Having Uncertain Functions with Unknown Bounds. *Proceeding of the American Control Conference,* Maryland., pp. 73,74.

[15] Yaz E. E., A. Azemi (1993).Sliding mode observer for nonlinear models with Unbounded Noise and Measurement Uncertainties., *J. Dynamics and Control*, Netherlands,Vol. **3**, pp.217-235.

[16] Yoo D. S. , M. J. Chung (1992). A variable structure control with simple adaptation laws for upper bounds on the norm of the uncertainties. *IEEE Trans. Autom Control*, Vol.**37**, pp. 860-865.