Heterarchical Distributed State Estimation and Failure Detection in Dynamic Systems

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***Abstract*— In this paper we discuss a class of failure detection techniques that is based on the heterarchical distributed state estimation structure. A heterarchical structure is defined as a data processing system in which there is no explicit hierarchy. Within this discussion, we present a distributed sensor fault detection and isolation method that results from merging hierarchical state estimation theory with the overlapping decompositions, expansions, and stability of large scale systems theory. The benefits of such a discussion include to have design methodologies of detection, isolation and diagnosis of sensor faults, that admits a range of implementations, allowing a tradeoff study of system complexity vs. performance.**

Index Terms – ***Kalman filters, Hierarchical Structures, Estimation Theory, Distributed Models, Hierarchical Control, Fault Detection, Overlapping Decompositions, Sensor Failures, Sensors.***

I. INTRODUCTION

An HETERARCHICAL approach may be the feasible way to health monitor large-scale systems since it decomposes the problem down into potentially smaller local problems. These local results can be blended into a global result that describes the health of the entire system. The benefits of such an approach include added fault tolerance and easy scalability.

State estimators are, upon many aspects, an useful tool for fault detection and identification (FDI). Failures act as unexpected inputs into a system and, thus, drive the error residual of any state estimation to biased values. With careful selection of the state estimation structure, these fault-driven residuals can be made to have persistent and distinctive characteristics. In many cases, freedom exists to address other design issues, such as noise sensitivity and parameter robustness. For these reasons, the application of state estimators to the problem of fault detection and identification has long been an rich area of research.

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In this paper, we will look at some of the challenges inherent to detecting faults in large-scale systems. complete description of a complex system and its effective control requires a great quantity and variety of sensors. For such systems, a heterarchical fault detection scheme may be the logical approach to the problem.

The heterarchical fault detection method is the result of combining the hierarchical state estimation theory studied in [2] and extended in [14] with the decentralized control theory developed by [16]. It compares the overlapped state estimates provided from heterarchical distribute state estimators each driven by independent measurement sets.

The remainder of the paper is organized as follows. In section II, the heterarchical state estimation structures to be utilized as basis to develop the fault detectors are delineated. Essential insights reveal that heterarchical state estimators are much more suited for distributed state estimation than hierarchical state estimators that do not. An important part of section II is how to obtain the global/local decomposition needed to develop the basis of the fault detector and isolator. This lead us to develop a heterarchical fault detection method based upon overlapping decomposition. We describe this method in section III, based on one specific heterarchical distributed state estimation structure discussed in section II.

II. STRATEGIES OF HETERARCHICAL DISTRIBUTED STATE ESTIMATION

The general theory of hierarchical systems was and is continuing to be applied to control and estimation. This application involves optimization techniques – minimum variance in the Kalman approach – and concepts of hierarchical structures. The aim is to construct state estimation architectures with different performance degrees.

We present and analyze the dynamics of the hierarchical structures to yield distributed state estimation methods.

The problem of heterarchical distributed state estimation formulated in this work deals exactly with the decomposition of the correction stage and is based on the original version of the Kalman filter [6] as well as on its alternative Inverse Covariance form[1].

A. *Strategies via Matrix Partitioning*

These strategies are based on the Inverse Covariance of the Kalman filter [1].

Consider the global system model:

$$x_{k+1} = A_k x_k + w_k \tag{1}$$

where w_k is independent of x_0 assumed Gaussian with covariance P_0.

In addition, consider a set of N local observations concerning the global system (1), comprised by the following equations:

$$y_k^i = H_k^i x_k + v_k^i, \; i = 1,2,..., N \tag{2}$$

where the v^i , measurement noises, with covariance R^i_k, are independent among themselves and independent of w_k and x_0.

For the heterarchical distributed state estimation problem we assume that the local processing algorithms are solved based on local models described by:

$$x_k^i = A_k^i x_k^i + w_k^i \tag{3}$$

$$z_k^i = C_k^i x_k^i + v_k^i \tag{4}$$

where i=1,2,..,N.

Consider the global system with the state x, decomposed into two subsystems with states x^1 and x^2. The local observations, y^1 and y^2 , in (1)-(2), based on the knowledge of x, provide an exact representation of the process. On the other hand, the models describing the local subsystems states, x^1 and x^2 , of the global system x, and the local observations based on knowledge of x^1 and x^2 , in (3)-(4), could provide only an *approximate* representation of the global system state x.

The global state estimation that will be processed in a centralized node, based on the observation of the global system (2), can be written as follows:

$$\hat{x} = \bar{x} - P\sum_{i=1}^{N} H^{i^t} R^{i^{-1}} H^i \bar{x} + P\sum_{i=1}^{N} H^{i^t} R^{i^{-1}} y^i \tag{5}$$

where $\bar{x} \equiv$ prediction of x.

$P \equiv$ covariance of the estimation error of x.

If there is a transformation T^i that satisfies the relationship between the local and global dynamics such that the measurements y^i and z^i in (2) and (4) become exactly or approximately compatible, then processing at the local nodes solves the following local estimation problem:

$$H^{i^t} R^{i^{-1}} y^i = \Gamma^{i^t} P^{i^{-1}} [\hat{x}^i - (I - H^{i^{-1}} . \\ .R^{i^{-1}} H^i \Gamma^i)\bar{x}^i] \tag{6}$$

where Γ^i is the nodal transformation matrix that satisfies $\Gamma^i = C^{i^\#} . H^i$ and # denotes the pseudo-inverse.

From (5) and (6) we have:

$$\hat{x} = \Lambda\bar{x} + \sum_{i=1}^{N} G^i (\hat{x}^i - \Lambda^i \bar{x}^i) \tag{7}$$

where

$$\Lambda = I - P\sum_{i=1}^{N} H^{i^t} R^{i^{-1}} H^i;$$
$$G^i = P\mathrm{T}^{i^t} P^{i^{-1}}; \tag{8}$$
$$\Lambda^i = I - P^i H^{i^t} R^{i^{-1}} H^i \Gamma^i;$$

In general, the local estimates $\hat{x}^i$ are not independent. The correlation between these estimates is taken account through the P matrix. The local correction gain G^i given in (8) incorporates the influence of these correlations in the global estimation process represented in (7).

If there is a nodal transformation T^i in (8) that transforms the global model in a feasible local model, such that P, for example, be diagonalizable, then we can construct an heterarchical and suboptimal global estimator in order to *undo the hierarchy*.

In principle, the strategies via matrix partitioning [2] and [4], as well as the strategies via the multiple projections [5], presented in the following subsection, require centralizer and coordinator modules, respectively, in order to fuse the local estimates in such hierarchical estimation structure.

B. *Strategies via Multiple Orthogonal Projections*

These strategies are based on the original version of the Kalman filter [6]. In this class of strategies each local node disposes only of its local model that represents exactly a subsystem of the global system. Therefore, the construction of heterarchical distributed structures based on these strategies assumes the existence of a nodal transformation T^i , not explicit, that satisfies exact relationships between the global system and the local subsystems.

From this assumption results the requirement of a coordinator module in order to give support to the local estimates processing.

Consider the following representations for the local models:

$$x_{k+1}^i = A_k^i x_k^i + \sum_{\substack{i=1 \\ j \neq i}}^{N} A_k^{ij} x_k^j + w_k^i$$
$$y_k^i = H_k^i x_k^i + v_k^i \qquad (9)$$

where the same assumptions made to the noise variables in (1) and (2) are held.

The key idea of the multiple projections method consists in the decomposition of the correction stage of the Kalman filter through the orthogonal projection of the state x^i on the observation vector of the global system. The observation vector is partitioned into N components of local observations. In this way, the following estimation result is obtained:

$$\hat{x}^i = \bar{x}^i + \sum_{i=1}^{N} E(x^i / \tilde{y}_i^{i-1}) \qquad (10)$$

where

$\sum_{i=1}^{N} \tilde{y}_i^{i-1}$ generates the Hilbert subspace:

$$\tilde{y}^1_{(k/k-1)} \oplus \tilde{y}^1_{2(k/k)} \oplus \tilde{y}^2_{3(k/k)} \oplus \ldots \oplus \tilde{y}^{N-1}_{N(k/k)}; \qquad (11)$$

$\bar{x}^i = E(x^i / \mathrm{Y}_{k-1})$;

$\mathrm{Y}_{k-1} \equiv$ observation subspace until the (k-1) instant.

The corrections based on the (N-1) nonlocal innovations, described by (10), constitute the coordinated hierarchical nature of the Kalman filter. In this hierarchical structure the important task of incorporating the inherent correlations among the local models, *a priori* partitioned exactly, and the global model, is made by the coordinator. In this way, the optimality of the estimation with coordinated hierarchy, in the Kalman sense, is preserved.

C. Discussions

The bottleneck in processing for hierarchical structures is caused by the centralizer or by the coordinator fusion of the information originating in the lower levels.

Discussions about this point have been made, e.g., in [2]-[5], [7] –[13], and [14].

In principle, (7) and (10) can be seen as global solutions to the hierarchical state estimation problem based on the dichotomy among the information filter and the state space Kalman filter representations. Using (7) and (10) as starting points, a synthetic diagram proposed as support to the development of distributed structures is shown in Fig. 1.

Distribution strategies based on the multiple projections method as well as on matrix partitioning, lead us to face the question on which model of the local subsystems to adopt considering the global model?

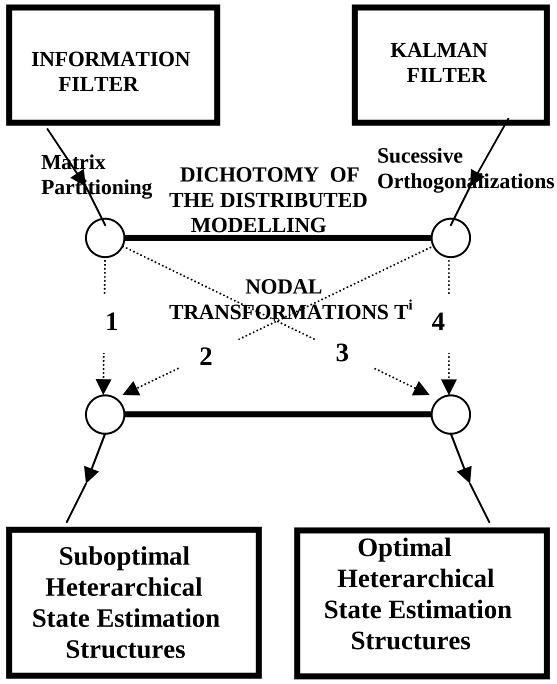


Fig. 1. Classes of Strategies for Heterarchical Distributed State Estimation

For distributed filters derived via matrix partitioning and the successive orthogonalizations, the nodal transformation matrices, under certain assumptions, can be implicitly modeled in such a way that, the local estimates can be considered *very close* to the optimal estimation.

In [9] a suboptimal state estimation structure is proposed via an analytical development. This structure is conformed in the class 2 of Fig. 1. of strategies, and its development is based on the hierarchical structure proposed in [5]. This analytical development and the form of the *approximate* nodal transformation used in [9] is based on the SPA (Supplementing Partitioning Approach) technique proposed in [11]. Also, in [8] a theorem that establishes the necessary and sufficient conditions to obtain the heterarchical distributed structure is presented, as well as for the analysis of the conditions heuristically established in [11].

The classes of state estimation strategies shown in Fig. 1 are, in details, discussed in [16].

III. System Model, Overlapping Decomposition and Fault Detection Method

Consider a large-scale linear interconnected system S, which is described by the following state and output equations:

$$\text{S}: x_{k+1} = Ax_k + w_k \tag{12}$$

$$y_k = H_k x_k + v_k \tag{13}$$

where $x \in R^n$, $w_k \in R^n$ is the state noise vector, $y_k \in R^m$ is the output measurement vector and $v_k \in R^m$ is the noise disturbing the output. A and H are the system matrices of appropriate dimensions, in which H is assumed to be a block-diagonal matrix with N blocks corresponding to N subsystems.

For the above system given by eqns. (12) and (13), we have the following assumptions:

(1) w_k and v_k are Gaussian random vectors with zero mean and covariances respectively given by $E\{w_j.w_k^t\}=Q\delta_{jk}$, $E\{v_j.v_k^t\}=R\delta_{jk}$

(2) The disturbance vectors are uncorrelated, i.e., $E\{v_j.w_k^t\}=0\ \forall\ j,k$

(3) The initial state vector x(0) is a Gaussian random vector with mean $E\{x(0)\}=X_0$ and covariance $E\{[x(0)-X_0][x(0)-X_0]^t\}=P_0$

(4) x(0) and the noise vectors v_k and w_k are uncorrelated, i.e., $E\{x(0).v_k^t\}=0$, $E\{x(0).w_k^t\}=0\ \forall\ k$

The system S described by equations (12) and (13) can be expanded into another system $\underline{\underline{S}}$ using a linear transformation

$$\underline{\underline{x}}_k = Tx_k \tag{14}$$

where $\underline{\underline{x}} \in R^{\underline{\underline{n}}}$ ($\underline{\underline{n}}>n$) and T is a $\underline{\underline{n}}$xn constant transformation matrix. The expanded system is given by:

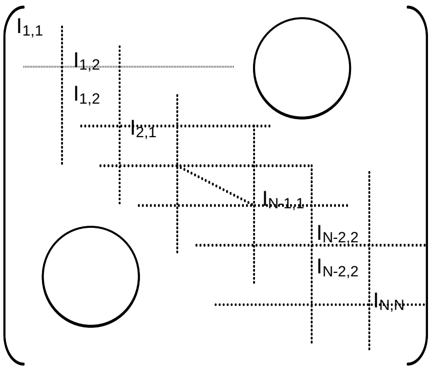
$$\underline{\underline{S}}: \underline{\underline{x}}_{k+1} = \underline{\underline{A}}\,\underline{\underline{x}}_k + \underline{\underline{w}}_k \tag{15}$$

$$y_k = \underline{\underline{H}}\,\underline{\underline{x}}_k + v_k \tag{16}$$

where $\underline{\underline{w}}_k$ is the expanded state noise and $\underline{\underline{A}}$ and $\underline{\underline{H}}$ are the new system matrices (with dimensions $\underline{\underline{n}}x\underline{\underline{n}}$, mx$\underline{\underline{n}}$ respectively) given by:

$$\underline{\underline{A}} = TAT^I + M;\ \underline{\underline{H}} = HT + L \tag{17}$$

$$T^I = (T^tT)^{-1}T \tag{18}$$

T = 

(8)

where $I_{i,1}$ is an identity matrix with dimension $(n_i-n_{i,2})$ x $(n_i-n_{i,2})$; $I_{i,2}$ is an identity matrix with dimension $n_{i,2}$ x $n_{i,2}$, i=1,...,N.

In this section, we consider the problem of detecting the malfunctioning sensors of the augmented system $\underline{\underline{S}}$, which comprises N overlapping subsystems. This will be carried out through the design of Partially Decentralized Hierarchical Kalman Filters, presented in [9] for the subsystems and by comparing the estimated states, which are obtained by two successive filters for each subsystem.

The ith subsystem $\underline{\underline{S}}_i$ derived from the expansion is described by the following equations:

$$\underline{\underline{S}}_i: \underline{\underline{x}}^i_{k+1} = \underline{\underline{A}}^i_k\,\underline{\underline{x}}^i_k + \sum_{\substack{j=1\\ i\neq j}}^{N} \underline{\underline{A}}^{ij}_k\underline{\underline{x}}^j_k + \underline{\underline{w}}^i_k \tag{20}$$

$$y^i_k = \underline{\underline{H}}^i_k\underline{\underline{x}}^i_k + v^i_k \tag{21}$$

The results obtained for partially decentralized hierarchical state estimation [9] can be applied by duality to the overlapping subsystems (20) and (21). The filters are designed as follows:

Consider the approximate equation of the expanded subsystem $\underline{\underline{S}}_i$:

$$\underline{\underline{S}}_i: \underline{\underline{x}}^i_{k+1} = \underline{\underline{A}}^i_k\,\underline{\underline{x}}^i_k + \sum_{\substack{j=1\\ i\neq j}}^{N} \underline{\underline{A}}^{ij}_k\underline{\underline{x}}^j_{k/k} + \underline{w}^i_k \tag{22}$$

$$y^i_k = \underline{\underline{H}}^i_k\underline{\underline{x}}^i_k + v^i_k \tag{23}$$

where

$$\underline{w}^i_k = \sum_{\substack{j=1\\ i\neq j}}^{N} \underline{\underline{A}}^{ij}_k\,\chi^j_k + \underline{\underline{w}}^i_k \tag{24}$$

with $\chi^j_k = \underline{\underline{x}}^j_k - \underline{\underline{x}}^j_{k/k}$

By using (24) as a "plausible" approximation [9] to represent a white noise, we can estimate the state $\underline{x}^i_{k+1}$, using a set of partially decoupled Kalman filters [9] described by the following stages.

Prediction Stage

$$\underline{x}^i_{k+1/k} = \underline{\underline{A}}^i_k\,\underline{\underline{x}}^i_{k/k} + \sum_{\substack{j=1\\ i\neq j}}^{N} \underline{\underline{A}}^{ij}_k\,\underline{\underline{x}}^j_{k/k} \tag{25}$$

$$\underline{P}^i_{k+1/k} = \underline{\alpha}^i_{k+1/k}\underline{\underline{A}}^{i\ t}_k + \underline{Q}^i_k \tag{26}$$

where

$$\underline{\alpha}^i_{k+1/k} = \underline{\underline{A}}^i_k\underline{P}^i_{k/k} \tag{27}$$

$$\underline{Q}^i_k = \underline{Q}^i_k + \underline{\alpha}^{ij}_{k+1/k} \tag{28}$$

$$\underline{\alpha}^{ij}_{k+1/k} = \sum_{\substack{j=1 \\ i \neq j}}^{N} \underline{A}^{ij}\underline{P}^{j}_{k/k}\underline{A}^{ij\,t}_{k} \quad \textbf{(29)}$$

$\underline{Q}^{i}_{k}$ Covariance matrix of the $\underline{w}^{i}_{k}$ approximate expanded white noise;

$\underline{P}^{i}_{k/k},\underline{P}^{j}_{k/k}$ Covariance matrices of the ith and jth approximate expanded subsystems, respectively.

Correction Stage

$$\underline{x}^{i}_{k/k} = \underline{x}^{i}_{k/k-1} + G^{i}_{k}\,\gamma^{j}_{k/k-1} \quad \textbf{(30)}$$

where

$$G^{i}_{k} = \underline{P}^{i}_{k/k-1}\,\underline{H}^{i\,t}_{k}\,(\,\underline{H}^{i}_{k}\,\underline{P}^{i}_{k/k-1}\,\underline{H}^{i\,t}_{k} + R^{i}_{k}\,)^{-1} \quad \textbf{(31)}$$

denotes the gain matrices of the local Kalman filters and $\gamma^{j}_{k/k-1}$ is the measurement prediction error of the ith approximate expanded subsystem.

The covariance matrix of $\underline{x}^{i}_{k/k}$, based on $\gamma^{i}_{k/k-1}$ can be written as:

$$\underline{P}^{i}_{k/k} = K^{i}_{k}\,\underline{P}^{i}_{k/k-1} \quad \textbf{(32)}$$

$$K^{i}_{k} = I - G^{i}_{k}\,\underline{H}^{i}_{k} \quad \textbf{(33)}$$

Due to the approximation (24), the prediction correction based on the non local observations is unnecessary as demonstrated in [9].

Owing to overlapping decomposition, the state vectors $\underline{x}^{i}$ and $\underline{x}^{i-1}$ share the part $\underline{x}^{i-1,2}$, i.e. , $\underline{x}^{i-1} = [\,\underline{x}^{i-2,2}\ \ \underline{x}^{i-1,1}\ \ \underline{x}^{i-1,2}\,]^{t}$ and $\underline{x}^{i} = [\,\underline{x}^{i-1,2}\ \ \underline{x}^{i,1}\ \ \underline{x}^{i,2}\,]^{t}$.

Let $[\,\underline{x}^{i-1,2}_{k/k}\,]_{ss1}$, $[\,\underline{x}^{i-1,2}_{k/k}\,]_{ss2}$ represent the estimated values of the state vector $\underline{x}^{i-1,2}$ from the filters of subsystems i-1 and i, respectively.

For the case of two subsystems, during normal operation of the overall system we have

$$z_{1,2} = E\,(\,[\,\underline{x}^{1,2}_{k/k}\,]_{ss1} - [\,\underline{x}^{1,2}_{k/k}\,]_{ss2}\,) = 0 \quad \textbf{(34)}$$

where E is the mathematical expectance.

If one or more than one of the N subsystem sensors are malfunctioning, the above condition will be violated, as shown in Table 1.

As a result, $Z_{1,2}$ becomes biased (positive or negatively) because the discrepance between estimates of the corresponding overlapping state.

Thus, by examining $Z_{1,2}$, the faulty sensors can be localized as shown in the voting decision Table 1.

Table 1. Sensor fault decision table for two sensors

$z_{1,2}$		Sensor fault decision
negative	$\leq \varepsilon$	fault in y_1
	$> \varepsilon$	faults in y_1 and y_2
positive		fault in y_2
null		normal operation

This tolerance value ε that means the magnitude of the departure from zero-mean must be found for a specific application depending on noise considerations and model parameter uncertainty.

ε is a constant which is usually determined by the experience of the designer. However, the reability of the SFD scheme must be investigated in failure cases which are differents from those considered in obtaining the value of ε.

It is important to observe that such investigation can open perspectives to treat the failure estimation problem too, because different values of ε could be useful to characterize the failures.

The failure estimation problem involves the determination of the extent of failure that could be expressed by a sensor become completely non-operational (and "off" or "hard-over" failures), or it may simply suffer degradation in the form of a bias or increased inaccuracies, which may be modeled as abrupt changes in H matrix or increase in the sensor covariance as well.

Thus, by inspecting the validity of eqn.(34), we could not only detect the sensor failures among the N subsystems, but also know which once have failed.

It is important to highlight that the use of decentralized estimation (25-33) modified the SFD scheme originally proposed in [17] which uses differences between overlapping states of the subsystems, by the generation of different failure test conditions (Table 1). Another point to be noted is that in spite of not obtaining the best state estimate of $\underline{S}$, the unbias property is preserved, meaning that the scheme above will not only be useful as a composite fault detector but also as a good state estimator by using the inverse transformation of similarity.

Although from the point of view of the sensors output, the subsystem estimators are completely decoupled, by the fact the state corrections are based on purely located observations. In other words, such state corrections don't take into account the successive orthogonalizations between the subsystems. On the other hand, these estimators take into consideration the interaction terms between the subsystems.

If the interactions between the subsystems are strong (i.e. strongly connected subsystems), a malfunction in any sensor could affects all the local filter estimates, and by consequence, compromises the reliability of the SFD scheme proposed.

Thus, it remains to show that the SFD scheme proposed also works satisfactorily to systems where the interactions may be strong, due to the fact the approximation (24) be just considered "acceptable" for weakly coupled systems [9].

Figure 2 illustrates the use of the decentralized state estimators to detect faulty sensors when subsystems $\underline{S}_1$ and $\underline{S}_2$ share the state variable $\underline{x}^2$.

In order to minimize the effect of noise, $z_{1,2}$ is passed through a low-pass filter such as:

$$z^{f}_{1,2}(k+1) = z^{f}_{1,2}(k) + g.[\, z_{1,2}(k+1) - z^{f}_{1,2}(k)] \qquad (35),$$

where g is the filter gain to be equally to ε chosen by simulations. The g gain attends on exclusively to smooth the estimator oscillations influenced by the state and measurement noise variances. In addition, if the state and measurement noise covariance matrices, diagonal Q and R, respectively, are such that all the elements q_{ij} are identical fpr all i=j and r_{ij} are identical fpr all i=j, then a unique gain g will smooth all the state estimation variable simultaneously.

The filtered output $z^{f}_{1,2}$ is used to measure the departure of $z_{1,2}$ from zero-mean, and thus to locate the faulty sensors.

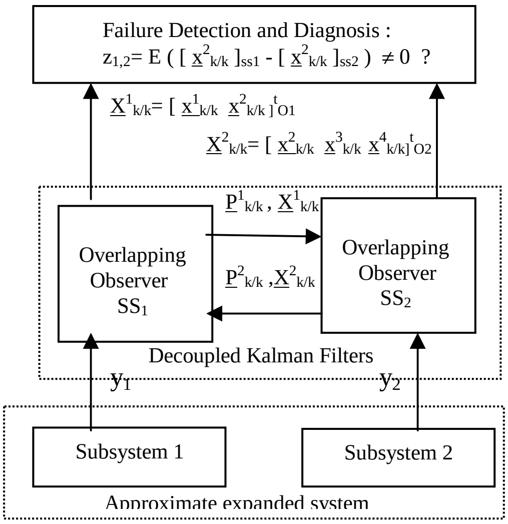


Figure 2. Approximate expanded system connected to failure detection system

IV. CONCLUSIONS

In this paper, methodologies in order to develop fault detectors based on heterarchical distributed state estimation structures are discussed. In addition, an extension of the Sensor Fault Detection method introduced in [17] has been proposed. The objective of the extension was to detect and isolate precisely composite sensor malfunctioning.

This is achieved by using an approximation which provides estimated interactions between the subsystems as portion of system noise.

Heterarchical distributed state estimators have been used to estimate the states of the overlapping subsystems and a procedure to incorporate new interactions within the filter equations has been described.

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