Revision on the Strict Positive Realness

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***Abstract* — In this paper, the necessary and sufficient conditions for strict positive realness of the real rational transfer functions are studied directly from basic definitions in the frequency domain. This paper deals with a new frequency domain approach to check if a real rational transfer function is a strictly positive real or not. This approach is based on the Taylor expansion and the Maximum Modulus Principle which are the fundamental tools in the complex analysis. Four related predominant statements in the strict positive realness area which is appeared in the control literature are discussed; the weaknesses and the drawbacks of these predominant statements are analyzed through some counter examples. Then a new necessary condition for strict positive realness are extracted via high frequency behavior of the Nyquist diagram. Finally the most simplified and completed conditions for strict positive realness are presented based on the complex analysis.**

Index Terms* — *Strict positive realness, frequency domain definitions, Taylor expansion approach, high frequency behavior, Maximum Modulus Principle.

I. INTRODUCTION

The concept of positive realness is motivated from circuit theory. The sufficiency condition for positive realness and many of it's properties are developed by Otto Brune in 1930 [1-2]. In 1963, Popov introduced the notion of hyperstability in control theory and showed that a linear time-invariant system is hyperstable system if and only if the transfer function of system is positive real, Also he developed the concept of strict positive realness and showed that a linear time-invariant system is asymptotically hyperstable system if and only if the transfer function of system is strictly positive real [3]. Thus the concept "strict positive realness" of transfer functions has been extensively used in various field of control such as Adaptive control [8-10], Optimal control [11-12], Nonlinear control [13-15], Robust control [16-21] and even Intelligent control [22]. The basic definition of strict positive realness is motivated by Popov's hyperstability theory which is stated in frequency domain, but it seems that the frequency domain tools achieved less attention and almost all activities are focused on the state space approaches, specific Kalman-Yakubovich-Popov (KYP) lemma [23-29]. With expiry fourth decade, still there is not unique statement which states the necessary and sufficient frequency domain conditions for strict positive realness in the control literature. In this paper, the Taylor expansion approach are introduced and used for study four predominant statements in this area. Then new necessary conditions which imposed by high frequency behavior of the Nyquist diagram are extracted and finally the most simplified and completed conditions in frequency domain for strict positive realness are presented based complex analysis.

II. BASIC DEFINITIONS

Let $G(s)$ denote a rational transfer function with real coefficients, then we have following definitions.

Definition 2.1 [7]*:* $G(s)$ is positive real (PR) if and only if:
1) $G(s)$ is analytic in $\text{Re}[s] > 0$,
2) Any pure imaginary pole of $G(s)$ is a simple pole and the associated residue is positive,
3) For all real $\omega \geq 0$ for which $j\omega$ is not a pole of $G(s)$, the inequality $\text{Re}[G(j\omega)] \geq 0$ is satisfied.

Definition 2.2 [3]*:* $G(s)$ is strictly positive real (SPR) if $G(s - \varepsilon)$ is PR for sufficiently small $\varepsilon > 0$.

The fundamental question which the four predominant statements in SPR area are trying to answering it, is: Which extra conditions must be hold on an PR transfer function to have an SPR transfer function? An important result of paper will be ability to specify answer of this question.

III. TAYLOR EXPANSION APPROACH

Suppose $G(s)$ in (3.1), be a real rational transfer function of the complex variable $s=\sigma+j\omega$.

$$G(s) = k\,\frac{b(s)}{a(s)} = k\,\frac{s^m + b_1 s^{m-1} + \ldots + b_m}{s^n + a_1 s^{n-1} + \ldots + a_n}\ ,\ k \neq 0 \qquad (3.1)$$

An important result implies from basic definitions in previous section, can be stated as following Lemma:

Lemma 3.1: $G(s)$ is SPR if and only if:
1) $G(s)$ is analytic in $\text{Re}[s] \geq 0$,
2) $n - m \geq -1$ and if $n - m = -1$ Then $k > 0$
3) $\text{Re}_{\varepsilon\to 0^+}[G(j\omega - \varepsilon)] \geq 0$, $\forall \omega \geq 0$.

Proof: According to the basic definitions, $G(s)$ is SPR if and only if the following conditions for sufficiently small $\varepsilon > 0$ are satisfied:
1) $G(s - \varepsilon)$ is analytic in $\text{Re}[s] > 0$,
2) Any pure imaginary pole of $G(s - \varepsilon)$ is a simple pole and the associated residue is positive,
3) For all real $\omega \geq 0$ for which $j\omega$ is not a pole of $G(s - \varepsilon)$, the inequality $\text{Re}[G(j\omega - \varepsilon)] \geq 0$ is satisfied.

The phrase "$G(s - \varepsilon)$ is analytic in $\text{Re}[s] > 0$" is equivalent to the phrase "$G(s)$ is analytic in $\text{Re}[s] > -\varepsilon$". It is obvious that a real rational transfer function of complex variable $s=\sigma+j\omega$ is analytic in the whole complex plane except in its poles, now suppose $G(s)$ be analytic in region $\mathrm{Re}[s] \geq 0$ and the nearest pole to the imaginary axis has a real part equal to $-p^*$, we can always select ε such that satisfied inequality $\varepsilon < p^*$. Thus the phrase "$G(s-\varepsilon)$ is analytic in $\mathrm{Re}[s] > 0$" is equivalent to the phrase "$G(s)$ is analytic in $\mathrm{Re}[s] \geq 0$". It is clear that the second condition restricts the relative degree of $G(s)$ and if the relative degree of $G(s)$ is equal to minus one then the positivity of k is necessary to guarantee positivity of associated residue for simple pole in infinity. It is clear that the third condition can be restated as appear in Lemma 3.1. □

We know that Taylor expansion of a rational transfer function $G(s)$ is valid on the whole complex plane except on the poles of $G(s)$. The first condition in the lemma 3.1, guarantee the validity of Taylor expansion of $G(s)$ on the imaginary axis, hence Lemma 3.1 can be restated as follows:

Lemma 3.2: $G(s)$ is SPR if and only if:
1) $G(s)$ is analytic in $\mathrm{Re}[s] \geq 0$,
2) $n-m \geq -1$ and if $n-m=-1$ Then $k>0$
3) $\lim_{\varepsilon\to 0^+} \mathrm{Re}[G(j\omega) - \varepsilon G'(j\omega) + \frac{1}{2!}\varepsilon^2 G''(j\omega) \mp \ldots] \geq 0$, $\forall\omega \geq 0$,

where $G^{(k)}(j\omega) = \left.\frac{d^k}{ds^k} G(s)\right|_{s=j\omega}$.

Suppose $G_e(s)=(1/2)[G(s)+G(-s)]$ and $G_o(s)=(1/2)[G(s)-G(-s)]$ are the even and odd parts of $G(s)$ respectively. Since the derivative of an even rational transfer function is an odd transfer function and the derivative of an odd rational transfer function is an even transfer function, hence it is easy to verify that:

$$\mathrm{Re}[G(j\omega-\varepsilon)] = G_e(j\omega) - \varepsilon G_o'(j\omega) + \frac{1}{2!}\varepsilon^2 G_e''(j\omega) \mp_e^o \ldots \qquad (3.2)$$

Thus the Lemma 3.2 can be restated as follows:

Lemma 3.3: $G(s)$ is SPR if and only if :
1) $G(s)$ is analytic in $\mathrm{Re}[s] \geq 0$,
2) $n-m \geq -1$ and if $n-m=-1$ Then $k>0$
3) $\lim_{\varepsilon\to 0^+} \left\{ G_e(j\omega) - \varepsilon G_o'(j\omega) + \frac{1}{2!}\varepsilon^2 G_e''(j\omega) \mp_e^o \ldots \right\} \geq 0$, $\forall\omega \geq 0$

IV. PREVIOUS PREDOMINANT STATEMENTS

In spite of the basic definition of SPR functions has been motivated by Popov's hyperstability theory and stated in frequency domain [3], it seems that the frequency domain tools achieved less attention and almost all activities is focused on the state space approaches, specific the Kalman-Yakubovich-Popov (KYP) lemma. In this section the four well-known predominant statements that state the necessary and sufficient conditions for SPR functions are discussed based on Lemma 3.3 as a criterion for strict positive realness which resulted directly from frequency domain definitions.

Theorem 4.1 [4, Astrom]: $G(s)$ is SPR if and only if:
1) $G(s)$ is analytic in $\mathrm{Re}[s] \geq 0$,
2) $G(s)$ has no any pole or zero on the imaginary axis,
3) $\mathrm{Re}[G(j\omega)] \geq 0, \forall\omega \geq 0$.

Counter Example 4.1: According to this theorem the transfer function $G_1(s) = \frac{s^2+s+1}{s^2+s+4}$ is SPR, because:

$$\mathrm{Re}[G_1(j\omega)] = G_{1e}(j\omega) = \frac{(\omega^2-2)^2}{(\omega^2-4)^2+\omega^2} \geq 0,\ \forall\omega \geq 0$$

But using Lemma 3.3 we have

$$\mathrm{Re}[G_1(j\omega-\varepsilon)] = \left.\frac{(s^2+2)^2}{(s^2+4)^2-s^2} - \varepsilon\left(\frac{-9s^4-21s^2+48}{\left((s^2+4)^2-s^2\right)^2}\right) \pm \ldots\right|_{s=j\omega}$$

$$= \frac{(\omega^2-2)^2}{(\omega^2-4)^2+\omega^2} - \varepsilon\left(\frac{-9\omega^4+21\omega^2+48}{\left((\omega^2-4)^2+\omega^2\right)^2}\right) \pm \ldots$$

Now it is easy to verify that

$$\mathrm{Re}[G_1(j\sqrt{2}-\varepsilon)] = -1.5\varepsilon + h.o.t(\varepsilon)$$

Thus $G(s)$ is not SPR by the basic definition because

$$\lim_{\varepsilon\to 0^+} \mathrm{Re}[G_1(j\sqrt{2}-\varepsilon)] = -1.5\varepsilon < 0.$$

Theorem 4.2 [5, Slotine]: $G(s)$ is SPR, if and only if:
1) $G(s)$ is analytic in $\mathrm{Re}[s] \geq 0$,
2) $\mathrm{Re}[G(j\omega)] > 0, \forall\omega \geq 0$.

Comment 4.1: It should be noted that, if r is the relative degree of $G(s)$, then the relative degree of $G^{(k)}(s)$ is $r+k$, hence the first two terms of the Taylor expansion are sufficient for study the behavior of $G(s)$ in sufficiently large frequencies. This fact will be used in counter example 4.2 and example 4.1 .

Counter Example 4.2: According to theorem 4.2 the transfer function $G_2(s) = \frac{s+1}{s^2+s+1}$ is SPR, because:

$$\mathrm{Re}[G_2(j\omega)] = G_{2e}(j\omega) = \frac{1}{(\omega^2-1)^2+\omega^2} > 0,\ \forall\omega \geq 0$$

But using Lemma 3.3 we have

$$\mathrm{Re}[G_2(j\omega-\varepsilon)] = \frac{1}{(\omega^2-1)^2+\omega^2} - \varepsilon\left(\frac{\omega^6+\omega^4-3\omega^2}{\left((\omega^2-1)^2+\omega^2\right)^2}\right) \pm \ldots$$

Thus $G(s)$ is not SPR according to the basic definitions because

$$\mathrm{Re}_{\omega\to\infty}[G_2(j\omega-\varepsilon)] \approx -\frac{\varepsilon}{\omega^2} < 0,\ \forall\varepsilon > 0$$

Theorem 4.3 [6, Ioannou and Tao]: $G(s)$ is SPR, if and only if :
1) $G(s)$ is analytic in $\mathrm{Re}[s] \geq 0$,
2) $\mathrm{Re}[G(j\omega)] > 0, \forall\omega \geq 0$,
3) One of the following conditions is satisfied:

a) If $n-m=-1$ Then: $\begin{cases} i)\ \lim_{\omega\to\infty} \mathrm{Re}[G(j\omega)]>0 \\ ii)\ \lim_{s\to\infty} \dfrac{G(s)}{s}>0 \end{cases}$,

b) If $n-m=1$ Then $\lim_{\omega\to\infty} \omega^2\ \mathrm{Re}[G(j\omega)]>0$.

Comment 4.2: The above theorem is correct, but it should be noted that the condition i) of a) of 3) in above theorem is not appear in [8-9]. Therefore the necessity of this condition is mentioned by following example.

Example 4.1: Suppose $G_3(s)=\dfrac{s^2+s+1}{s+1}$, according to Lemma 3.3 we have:

$$\mathrm{Re}[G_3(j\omega-\varepsilon)]=\frac{1}{\omega^2+1}-\varepsilon\left(\frac{\omega^4+3\omega^2}{(\omega^2+1)^2}\right)\pm ...$$

And using Comment 4.1 implies

$$\mathrm{Re}_{\omega\to\infty}[G_3(j\omega-\varepsilon)]\approx-\varepsilon<0$$

Thus G(s) is not SPR.

It is easy to show that all conditions in theorem 4.3 are satisfied for $G_3(s)$, except the condition i) of a) of 3).

Theorem 4.4 [7, Khalil]: Suppose $G(s)$ is a proper rational transfer function, then $G(s)$ is SPR, if and only if:

1) $G(s)$ is analytic in $\mathrm{Re}[s]\ge 0$,

2) $\mathrm{Re}[G(j\omega)]>0, \forall\omega\ge 0$,

3) One of the following conditions is satisfied:

a) If $n-m=0$ Then $k>0$

b) If $n-m=1$ Then $\lim_{\omega\to\infty} \omega^2\ \mathrm{Re}[G(j\omega)]>0$

This theorem states the conditions only for proper transfer functions and it is correct. In general, the third condition which is appeared in the two last theorems implies the fact that there are extra necessary conditions for SPR functions which are imposed by high frequencies. This fact is studied in next section using high frequency behavior of transfer functions in Nyquist diagram.

V. A New Necessary Condition

Suppose $G(s)$ is a real rational transfer function as shown in (3.1), it should be noted that the inequalities $k>0, |n-m|\le 1$ and $b_i\ge 0, i=1,...,m$; $a_j\ge 0, j=1,...,n$ can easily be resulted from circuit theory for any positive real function. Therefore assume that $k>0, |n-m|\le 1$ in (3.1), it is easy to show that if $n=m$, then there is not any extra condition which is imposed by high frequencies for strict positive realness because $G(s)\to k$ *as* $s\to\infty$. Now if $|n-m|=1$, then the extra condition $(n-m)(a_1-b_1)>0$, is imposed by high frequencies, this fact is explained in the following lemma.

Lemma 5.1: If $G(s)$ is SPR and $|n-m|=1$, then the inequality $(n-m)(a_1-b_1)>0$ will be satisfied.

Proof: Suppose $G(s)$ in (3.1) and $k>0, |n-m|\le 1$. Now if $n-m=1$, it is obvious that, if $G(s)$ is PR then its Nyquist diagram lies at closed right half complex plane and $G(s)\to k/(s+\alpha)$ as $s\to\infty$, thus the derivative of $\arg G(j\omega)$ can not be positive at sufficiently large frequencies. Also equality $\arg\{1/G(j\omega)\}=-\arg G(j\omega)$, implies that the derivative of $\arg G(j\omega)$ is not negative when $n-m=-1$, hence if $G(s)$ is PR and $|n-m|=1$, then the inequality

$$(n-m)\left(\frac{d}{d\omega}\arg G(j\omega)\right)_{\omega\to\infty}\le 0\ ,\ |n-m|=1 \tag{5.1}$$

must be satisfied. Now

$$G(j\omega-\varepsilon)=k\,\frac{(j\omega-\varepsilon)^m+...+b_m}{(j\omega-\varepsilon)^n+...+a_n}=k\,\frac{\prod_{i=1}^{m}(j\omega-\varepsilon-z_i)}{\prod_{l=1}^{n}(j\omega-\varepsilon-p_l)} \tag{5.2}$$

it is easy to see that

$$\arg G(j\omega-\varepsilon)=\sum_{i=1}^{m}\tan^{-1}\left(\frac{\omega-\mathrm{Im}\,z_i}{-\varepsilon-\mathrm{Re}\,z_i}\right)-\sum_{l=1}^{n}\tan^{-1}\left(\frac{\omega-\mathrm{Im}\,p_l}{-\varepsilon-\mathrm{Re}\,p_l}\right) \tag{5.3}$$

Hence

$$\begin{aligned}\frac{d}{d\omega}\arg G(j\omega-\varepsilon)=&\sum_{i=1}^{m}\left(\frac{-\varepsilon-\mathrm{Re}\,z_i}{(\varepsilon+\mathrm{Re}\,z_i)^2+(\omega-\mathrm{Im}\,z_i)^2}\right)\\&-\sum_{l=1}^{n}\left(\frac{-\varepsilon-\mathrm{Re}\,p_l}{(\varepsilon+\mathrm{Re}\,p_l)^2+(\omega-\mathrm{Im}\,p_l)^2}\right)\end{aligned} \tag{5.4}$$

Now, it is easy to verify that

$$\begin{aligned}\left.\frac{d}{d\omega}\arg G(j\omega-\varepsilon)\right|_{\omega\to\infty}&\approx\sum_{i=1}^{m}\left(\frac{-\varepsilon-\mathrm{Re}\,z_i}{\omega^2}\right)-\sum_{l=1}^{n}\left(\frac{-\varepsilon-\mathrm{Re}\,p_l}{\omega^2}\right)\\&=\frac{b_1-a_1+(n-m)\varepsilon}{\omega^2}\end{aligned} \tag{5.5}$$

Therefore the inequality $(n-m)\left(\frac{d}{d\omega}\arg G(j\omega)\right)_{\omega\to\infty}\le 0$, implies that $(n-m)(a_1-b_1)\ge\varepsilon>0$, and this completed the proof of the lemma. □

Remark 5.1: If $G(s)$ is PR and $|n-m|=1$, then the inequality $(n-m)(a_1-b_1)\ge 0$ will be satisfied.

Remark 5.2: If $G(s)$ is SPR and $|n-m|=1$ then by equation (5.5) and Lemma 5.1 can be shown that the $\frac{d}{d\omega}\arg G(j\omega)$ can not decay more rapidly than ω^{-2} as $|\omega|\to\infty$.

Comment 5.1: The restriction which is introduced in Remark 5.2 there is not exist for the PR functions, and it is an important difference between PR and SPR functions that be resulted from high frequency behavior of transfer functions in Nyquist diagram. In the other words, it is clear that: $-b_1$ is equal to the summation of the zeros of $G(s)$ and $-a_1$ is equal to the summation of the poles of $G(s)$, hence the third condition which is appeared in the theorems 4.3 and 4.4 can be replaced with: if $|n-m| = 1$, then $a_1 \neq b_1$. Also the new necessary condition which is stated in Lemma 5.1 for $G(s)$ with relative degree one can be interpreted as follows:

$$a_1 - b_1 = \left(\sum_{l=1}^{n} \mathrm{Re}[-p_l] \right) - \left(\sum_{i=1}^{m} \mathrm{Re}[-z_i] \right) > 0 \tag{5.6}$$

Comment 5.2: Suppose $G(s)$ has relative degree one and it is in the form of (3.1), then

$$\mathrm{Re}[G(j\omega)] = k\frac{\mathrm{Re}\{b(j\omega)a(-j\omega)\}}{a(j\omega)a(-j\omega)} = k\frac{(a_1-b_1)\omega^{2(n-1)}+\dots}{\omega^{2n}+\dots} \tag{5.7}$$

thus $\lim_{\omega\to\infty} \omega^2 \mathrm{Re}[G(j\omega)] = k(a_1-b_1)$, hence the condition If $n-m=1$ Then $\lim_{\omega\to\infty} \omega^2 \mathrm{Re}[G(j\omega)] > 0$ which is appear in the theorems 4.3 and 4.4 can be restated as: If $n-m=1$ Then $k(a_1-b_1) > 0$, but the second condition in these theorems guarantee the inequality $k(a_1-b_1) \geq 0$ for $n-m=1$, therefore this condition can be simplified to: If $n-m=1$ Then $a_1 \neq b_1$.

The following examples illustrate utilization of this necessary condition.

Example 5.1: Let

$$G_4(s) = \frac{(s+4)(s+6)}{(s+2)(s+3)(s+5)}$$

According to (5.6) $a_1 - b_1 = (2+3+5) - (4+6) = 0$, thus using Lemma 5.1 and Remark 5.1, resulted that $G_4(s)$ is not SPR but maybe PR. Fig. 1 shows that $G_4(s)$ is PR.

Example 5.2: Let

$$G_5(s) = \frac{s^2+s+1}{2s^3+2s^2+3s+2}$$

According to (5.6) $a_1 - b_1 = (2/2) - (1) = 0$, thus using Lemma 5.1 and Remark 5.1, resulted that $G_5(s)$ is not SPR but maybe PR. Fig. 1 shows that $G_5(s)$ is not PR.

Example 5.3: Let

$$G_6(s) = \frac{(s^2+6s+11)(s^2+5s+3)}{(s^2+3s+7)(3s^3+18s^2+5s+9)}$$

According to (5.6) $a_1 - b_1 = (3+18/3) - (6+5) = -2$, thus using Lemma 5.1 and Remark 5.1, resulted that $G_6(s)$ is not PR. Fig. 1 shows that $G_6(s)$ is not PR.

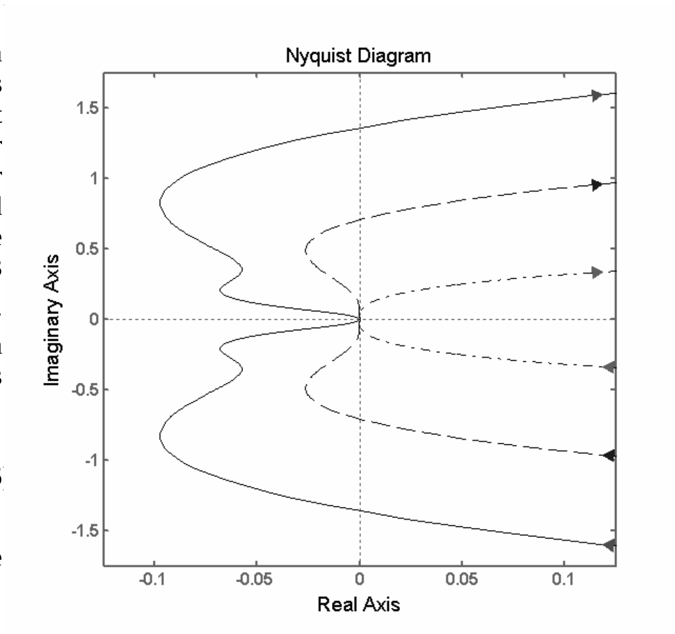

Fig. 1: $G_4(s)$: _._._._ , $G_5(s)$: __ __ __ , $G_6(s)$: ______

VI. Main Results

Theorem 6.1: $G(s)$ is SPR, if and only if :

1) $G(s)$ is analytic in $\mathrm{Re}[s] \geq 0$,
2) $n-m \geq -1$ and if $n-m=-1$ Then $k>0$
3) $\mathrm{Re}[G(j\omega)] > 0, \forall \omega \geq 0$
4) If the relative degree of $G(s)$ is nonzero then the summation of zeros and the summation of poles of $G(s)$ must be not equal, i.e. , if $|n-m|=1$ Then $a_1 \neq b_1$.

Proof: An important result implies from Maximum Modulus Principle in the complex analysis is that as follows:

Lemma 6.1 [30]: Suppose $G(s)$ is a function of complex variable $s=\sigma+j\omega$, now if it is analytic in a closed bounded region Γ and not constant throughout the interior of Γ, then $\mathrm{Re}[G(s)]$ has a minimum value in Γ which occurs on the boundary of Γ and never in the interior.

Now consider Lemma 3.1, the first condition states that $G(s)$ is analytic in $\mathrm{Re}[s] \geq 0$, therefore minimum value of $\mathrm{Re}[G(s)]$ occurs on the boundary $s = -\varepsilon + j\omega, \forall\omega \in R$ that appear in third condition of Lemma 3.1. If $\mathrm{Re}[G(j\omega)] \geq 0$ and exist finite frequency ω_0 such that $\mathrm{Re}[G(j\omega_0)] = 0$ then the above result of Maximum Modulus Principle implies that $\mathrm{Re}[G(j\omega_0 - \varepsilon)] < 0, \forall\varepsilon > 0$ and thus the inequality $\mathrm{Re}[G(j\omega)] > 0$ is necessary for $\mathrm{Re}_{\varepsilon\to 0^+}[G(j\omega-\varepsilon)] \geq 0, \forall\omega \in R$ to be satisfied. The fourth condition can be proved by study the inequality $\mathrm{Re}[G(j\omega-\varepsilon)] \geq 0, \forall\varepsilon > 0$ in the sufficiently large frequencies as discussed in the previous section. □

Comment 6.1 We know the inequalities $k > 0$, $|n-m| \le 1$ can easily be resulted from circuit theory for any positive real function, hence the above theorem can be restated to be more user-friendly as follows:

Theorem 6.2: The real rational transfer function

$$G(s) = k\,\frac{b(s)}{a(s)} = k\,\frac{s^m + b_1 s^{m-1} + ... + b_m}{s^n + a_1 s^{n-1} + ... + a_n}\ ,\ k \ne 0$$

is SPR, if and only if :

1) $k > 0$ and $|n-m| \le 1$ and if $|n-m| = 1$ Then $a_1 \ne b_1$,

2) $G(s)$ is analytic in $\mathrm{Re}[s] \ge 0$,

3) $\mathrm{Re}[G(j\omega)] > 0, \forall \omega \ge 0$

Comment 6.2: If $G(s)$ is PR, then it will be SPR if the following extra conditions are satisfied:

1) $G(s)$ has no any pole or zero on the imaginary axis,

2) $\mathrm{Re}[G(j\omega)] \ne 0, \forall \omega \in R$

3) If $|n-m| = 1$ Then $a_1 \ne b_1$

VII. Conclusion

In this paper, unlike other works which have focused on the state space tools such as KYP lemma, the results have been obtained directly from basic definitions in the frequency domain using complex analysis tools. The proposed method has been established based on the Taylor expansion and the Maximum Modulus Principle. Using Taylor expansion approach, the four predominant statements in the strict positive realness area has been studied. A new necessary condition based on the high frequency behavior of transfer functions has been also extracted and finally the most simplified and completed conditions for strict positive realness in frequency domain have been presented.

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