Nonlinear Predictive Adaptive Controllers For General Nonlinear Systems

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Abstract:
In this paper, we propose a new model reference adaptive predictive controller scheme for general non linear systems using analytic linearization. When the parameters used in the design of the predictive controller are unknown and time-varying, adaptive tools became very necessary in designing the controller. The adaptation of the parameters is calculated by the minimization of the predictive error in the receding horizon using a Taylor series expansion. To demonstrate the efficiency of the proposed scheme a comparison of the predictive algorithm with fixed gain elaborated by [1] and the adaptive proposed scheme are presented as illustrative example.

Index-terms: Nonlinear systems, Predictive control, adaptive control, model reference adaptive control, Feedback systems.

I. INTRODUCTION

Model predictive control (MPC) is now considered to be a popular nonlinear control method. Much extensive research, with considerable progress, has been made in MPC. One of this works has considered analytic linearization of nonlinear systems using a Taylor expansion to predict the error between future reference output and future output of the system. In [2-5] and [16] the authors presents the optimal predictive control of affine nonlinear systems. The big performances of this method are proven in [1-5]. However, this approach showed some failings if the parameters of the systems are unknown or variable during the time. In this way, in [6-8] a parameter updating algorithm based on the minimization of the predictive error is implemented in parallel with the control algorithm. The convergence and the efficiency of this algorithm are demonstrated in [7]. Following this matter, it is important to build up a predictive adaptive controller handling a more general nonlinear system which may be seen as a logic continuation of the work started in [1]. In order to solve the predictive control of a general nonlinear control system, we presents in this paper an extension of the work [6-8] to create an optimal adaptive predictive control law for the general nonlinear systems. This new control algorithm has been inspired from the works of [6-15].

This paper is organised as follows: in section II, a review of the optimal nonlinear predictive control algorithm of general nonlinear systems presented in [1] is summarised. While in section III, the new nonlinear predictive model reference adaptive algorithm is presented. In section VI, some simulation results are given showing the effectiveness of the proposed algorithm. A conclusion is presented in section V, giving some future work and comments.

II. REVIEW OF THE PREDICTIVE CONTROL OF GENERAL NONLINEAR SYSTEMS

Consider the following general non-linear single input single output (SISO) system:

$$\begin{cases} \dot{x}(t) & = & f(x(t),\, u(t)) \\ y(t) & = & h(x(t)) \end{cases} \tag{2.1}$$

Where $x \in \Re^n$ and $u, y \in \Re$ are the state, control and output respectively. The predictive non-linear control problem can be stated as follows (see [1]): At a given instant, based on the knowledge of the process past and present values of the output sequence and the past values of the input sequence, it is desired to determine the next control sequence in order that the process output rallies a set-point trajectory in the future without tracking error.

The optimal tracking problem is based on the idea that at any time *t*, to design within a moving time frame located at any time *t* regarding $x(t)$ as the initial condition of a state trajectory $\hat{x}(t+\tau)$ driven by an input $\hat{u}(t+\tau)$. The system dynamics in the moving horizon time frame located at time *t* are described by:

$$\begin{cases} \dot{\hat{x}}(t+\tau) & = f(\hat{x}(t+\tau)\ ,\ \hat{u}(t+\tau)) \\ \hat{y}(t+\tau) & = h(\hat{x}(t+\tau)) \end{cases} \tag{2.2}$$

Where $\tau \geq 0$ and the initial state vector $\hat{x}(t+\tau)$ in the moving time frame is given by: $\hat{x}(t) = x(t)$ (2.3)

The receding-horizon performance index adopted is given by:

$$J = \frac{1}{2}\int_{T_1}^{T_2}((\hat{y}(t+\tau)-\hat{y}_{ref}(t+\tau))^T(\hat{y}(t+\tau)-\hat{y}_{ref}(t+\tau))d\tau$$

(2.4)

Some assumptions are imposed on the non-linear system (2.1):

A1: The system is sufficiently differentiable with respect to time to any order;

A2: $f(0,0)=0$;

A3: $D_u D_{f_x}^k h(x)=0$. for $k=1,\dots,\mu-1$, and $D_u D_{f_x}^{\mu} h(x)\neq 0$. for all x and u;

Fig.1 shows a schematic configuration of the overall optimal predictive control.

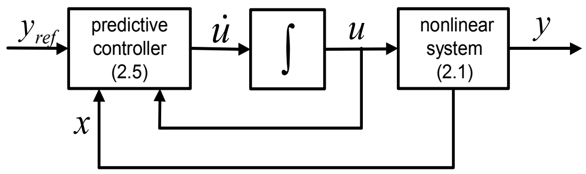

Fig.1: Block diagram of optimal predictive controller

As seen in Appendix A, the predictive controller can be written as:

$$\dot{u}(t) = -(D_u D_{f_x}^{\mu} h(x))^{-1}(KM_{\mu+1}(t)+D_{f_x}^{\mu+1}h(x)-y_{ref}^{[\mu+1]}(t)) \quad (2.5)$$

II. THE DESIGN OF THE ADAPTIVE OPTIMAL PREDICTIVE CONTROLLER

The following analysis will be based on the SISO system

$$\begin{cases} \dot{x} &= f(x,u,\theta) \\ y &= h(x,\theta) \\ x(0) &= x_0 \end{cases} \quad (3.1)$$

Where $x\in\Re^n$ is the state, $u, y\in\Re$ are the control and the output respectively and $\theta\in\Re^q$ is the vector of unknown parameters.

The general scheme of predictive adaptive control is outlined below in Fig.2

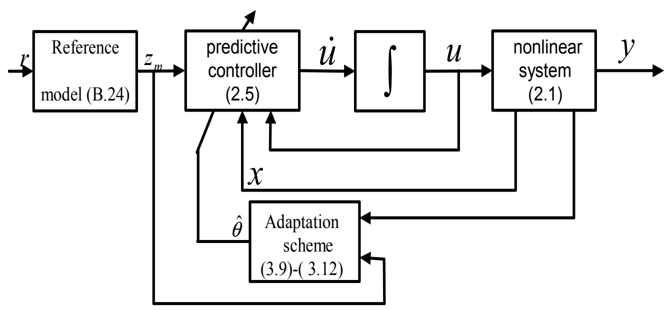

Fig. 2 Block diagram of optimal predictive adaptive control

The adaptation law consists as:

We define an error ξ as the difference between the predictive plant output $y(t+\tau)$ and a prescribed reference output $y_{ref}(t+\tau)$:

$$\xi = \int_{T_1}^{T_2} (y(t+\tau) - y_{ref}(t+\tau))\,d\tau \quad (3.2)$$

Where $y(t+\tau) = t(\tau)\ \hat{z}$ (3.3)

$y_{ref}(t+\tau) = t(\tau)\ z_m$ (3.4)

and $\hat{z} = \left[\hat{h}(x,\theta),\ D_{\hat{f}_x}\hat{h}(x,\theta),\dots D_{\hat{f}_x}^{\mu}\hat{h}(x,\theta)\right]^T$ (3.5)

$$z_m = [\ y_{ref},\ y^{(1)}_{ref},\ \dots\ y^{(\mu)}_{ref}\ ]^T \quad (3.6)$$

$$t(\tau) = \left(1,\ T_2 - T_1,\ \dots\ \frac{T_2^{\mu} - T_1^{\mu}}{(\mu)!}\right) \quad (3.7)$$

Therefore ξ can be written as

$$\xi = \int_{T_1}^{T_2} (t(\tau)\ (\hat{z}-z_m))\,d\tau = \int_{T_1}^{T_2} t(\tau)\,d\tau\ \ (\hat{z}-z_m) \quad (3.8)$$

The derivative of the augmented predictive error ξ_a is calculate in Appendix B and the result is given as :

$$\dot{\xi}_a = A'_n\,\xi_a + W'_1\Delta\theta + W'_2\,\dot{\hat{\theta}} \quad (3.9)$$

We introduce the signal "ε" satisfying

$$\begin{cases} \dot{\varepsilon} = A'_n\,\varepsilon + W'_2\,\dot{\hat{\theta}} \\ \varepsilon(0) = 0 \end{cases} \quad (3.10)$$

and define the augmented error $\eta(t)$ by $\eta = \xi_a - \varepsilon$ which satisfies $\dot{\eta} = A'_n\eta + W'_1\,\Delta\theta$ (3.11)

The form (3.11) is familiar in the linear adaptive control literature (with the difference being that in this case the regressor W'_1 is a non linear function of $x, \hat{\theta}$ and v) this form implies the parameter update law

$$\Delta\dot{\theta} = -\dot{\hat{\theta}} = -\Gamma^{-1} W_1'^T P\,\eta \quad (3.12)$$

Where $\Gamma>0$ the adaptation gain matrix and $P=P^T$ is a strictly positive matrix chosen to satisfy the Lyapunov equation $A_n'^T\,p + p\,A'_n = -I$ (3.13)

IV. SIMULATION RESULTS

Consider the second order general nonlinear system, with an input $u(t)$ and an output $y(t)$: (Note that this example is also treated in [1])

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) + \tanh(u(t)) \\ \dot{x}_2(t) &= -x_1(t) + \theta\,x_2^2(t) + u \\ y(t) &= x_1(t) \end{aligned} \quad (4.1)$$

Calculate the order μ of this system :

$$D_{f_x}h(x) = \frac{\partial h(x)}{\partial x} f(x,u) = x_2(t) + \tanh(u(t)),$$

$D_u D_{f_x} h(x) = \dfrac{1}{\cosh^2(u)} \neq 0$. Hence, $\mu = 1$.

When the predictive times are chosen as $T_1 = 1(s)$, the $T_2 = 2(s)$ and the order r for Taylor series expansion is chosen as $r=3$. The controller gain $K = [3.75\ 3]$ can be calculated by equation (A.9) into equation (A.11). Hence from (2.5), the derivative of the nonlinear predictive controller can be written as:

$$\dot{u}(t) = \frac{1}{\cosh^2(u)} \left(-\left(-x_1(t) + \hat{\theta} x_2^2(t) + u(t)\right) + y_{ref}^{[2]}(t) + k_2\left(\dot{y}_{ref}(t) - \dot{y}(t)\right) + k_1\left(y_{ref}(t) - y(t)\right) \right) \quad (4.2)$$

and assume

$$\begin{cases} \theta = 1 & \text{if } 0 \le t \le 30 \\ \theta = 5 & \text{if } 30 < t \end{cases} \quad (4.3)$$

We seek to construct a model reference adaptive controller for the plant (4.1) with a model

$$\dot{z}_m = \begin{pmatrix} 0 & 1 \\ -3.75 & -3 \end{pmatrix} z_m + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r \quad (4.4)$$

For an initial state $x_0 = [0 \quad 0]^T$, a reference input $r(t) = 4$, an adaptation gain $\Gamma = 0.002\, I$ and initial parameter estimate $\hat{\theta}(0) = 0$, a comparison was made between the controller with the adaptive scheme and a controller with fixed gains based on the initial parameter estimate. These result it shown in Fig.3 where $y_1(t)$ referred to not adaptive scheme and $y_2(t)$ referred to the adaptive scheme. The variation of the parameter $\hat{\theta}$ it shown in Fig.4

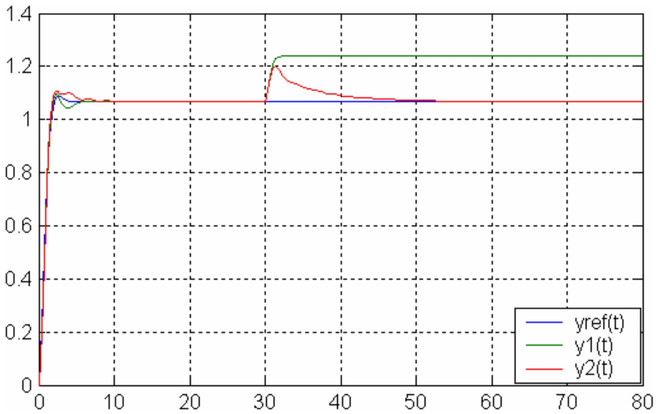

Fig. 3 outputs curves in non adaptive and adaptive settings

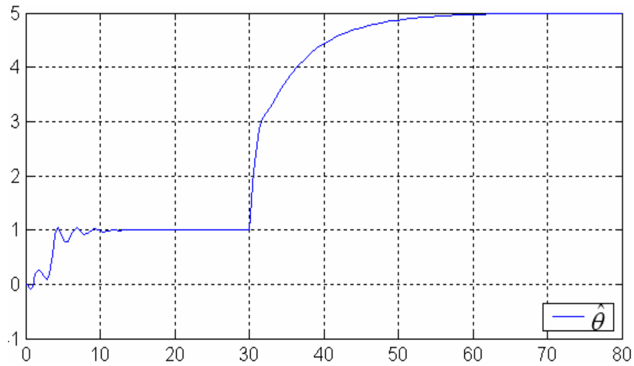

Fig.4 Variation of the parameter $\hat{\theta}$

V. CONCLUSION

In this paper, a new nonlinear predictive adaptive control algorithm is presented solving the problem of unknown or time varying parameters. Robustness and good performance of this new algorithm in term of tracking and regulation has been demonstrated by a several simulation examples. As perspective and future work, it will be interesting to develop a deep convergence analysis of this algorithm.

ACKNOWLEDGMENTS

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APPENDIX

A Optimal predictive control u*(t)*

1/ Output prediction

The moving time frame is predicted by Taylor series expansion. Repeated differentiation up to $\mu + 1$ times of the output $\hat{y}$ with respect to time, together with repeated substitution of the system (2.1) gives:

$$\dot{\hat{y}}(t) = \frac{\partial h(x)}{\partial x} \dot{x} = \frac{\partial h(x)}{\partial x} f(x,u) = D_{f_x} h(x)$$

$$\ddot{\hat{y}}(t) = \frac{\partial}{\partial x}\left(\frac{\partial h(x)}{\partial x} f(x,u)\right) \dot{x} + \frac{\partial}{\partial u}\left(\frac{\partial h(x)}{\partial x} f(x,u)\right) \dot{u} = D_{f_x}^2 h(x) \quad (A.1)$$

$$\vdots$$

$$\hat{y}^{[k]}(t) = D_{f_x}^k h(x) \quad \text{for } k = 3, \ldots, \mu$$

$$\hat{y}^{[\mu+1]}(t) = \frac{\partial D_{f_x}^{\mu}}{\partial x} f(x,u) + \frac{\partial D_{f_x}^{\mu}}{\partial u} \dot{u} = D_{f_x}^{\mu+1} h(x) + D_u D_{f_x}^{\mu} h(x) \dot{u}$$

Where:

$y^{[i]}(t)$ denotes the i th derivative of $\hat{y}(t)$, the relationship $\hat{x}(t+\tau) = x(t+\tau)$ for $\tau = 0$ is used.

When the control order is chosen as $r - \mu$, to make the ($r - \mu$)th derivative of the control signal appear, the order of the Taylor expansion of the output $\hat{y}(t+\tau)$ must be least r.

Differentiating the last equation in (2.5) with respect to time yields:

$$\hat{y}^{[\mu+2]}(t) = D_{f_x}^{\mu+2} h(x) + D_u D_{f_x}^{\mu} h(x) \ddot{u} + z_1(x,u,\dot{u}) \quad (A.2)$$

Where :

$$z_1(x,u,\dot{u}) = \frac{\partial D_{f_x}^{\mu+1} h(x)}{\partial u} \dot{u} + \frac{\partial D_u D_{f_x}^{\mu} h(x)}{\partial x} f(x,u) \dot{u} + \frac{\partial D_u D_{f_x}^{\mu} h(x)}{\partial x} \dot{u}^2 \quad (A.3)$$

Similarly it can be calculate the higher derivatives of the output until the r th-order, which is given by:

$$y^{[r]}(t) = D_{f_x}^{r} h(x) + D_u D_{f_x}^{\mu} h(x) u^{[r-\mu]} + z_{r-\mu-1}(x,u,\dot{u},\ldots,u^{[r-\mu-1]})$$

where $z_{r-\mu-1}(x,u,\dot{u},..., u^{[r-\mu-1]})$ is a complicated nonlinear function of $x, u, \dot{u},..., u^{[r-\mu-1]}$.
Collecting all derivatives yields :

$$\hat{\hat{Y}}(t) = \begin{bmatrix} \hat{y}^{[0]} \\ \hat{y}^{[1]} \\ \vdots \\ \hat{y}^{[\mu]} \\ \hat{y}^{[\mu+1]} \\ \vdots \\ \hat{y}^{[\mu+r]} \end{bmatrix} = \begin{bmatrix} h(x) \\ D_{f_x} h(x) \\ \vdots \\ D_{f_x}^{\mu} h(x) \\ D_{f_x}^{\mu+1} h(x) \\ \vdots \\ D_{f_x}^{r} h(x) \end{bmatrix} + \begin{bmatrix} 0^{(\mu+1)\times 1} \\ L(\hat{\bar{u}}) \end{bmatrix} \quad \text{(A.5)}$$

Where $L(\hat{u})$ is a matrix given as follows :

$$L(\hat{\bar{u}}) = \begin{bmatrix} D_u D_{f_x}^{\mu} h(x)\dot{u} \\ D_u D_{f_x}^{\mu} h(x)\ddot{u} + z_1(x,u,\dot{u}) \\ \vdots \\ D_u D_{f_x}^{\mu} h(x) u^{[r-\mu]} + \\ z_{r-\mu-1}(x,u,\dot{u},..., u^{[r-\mu-1]}) \end{bmatrix} \quad \text{(A.6)}$$

and

$$\hat{\bar{u}}(t) = \left[\hat{u}(t)^T \quad \dot{\hat{u}}(t)^T \quad \ddot{\hat{u}}(t)^T \quad .... \quad \hat{u}^{[r-\mu-1]}(t)^T \right]$$

Within the moving time frame, the output $\hat{y}\ (t+\tau)$ at the time τ is approximately predicted by $\hat{y}(t+\tau) = \Gamma(\tau)\ [\hat{y}^{[0]}, \hat{y}^{[1]},...., \hat{y}^{[r]}]^T$,

With $\Gamma(\tau) = [1, \quad \tau, \quad ...., \quad \frac{\tau^{(r)}}{(r)!}]$

Similarly $\hat{y}_{ref}(t+\tau)$ can be written as follow :

$$\hat{y}_{ref}(t+\tau) = \Gamma(\tau)\ \hat{\hat{Y}}_{ref}(t)$$

Where

$$\hat{\hat{Y}}_{ref}(t) = \left[y_{ref}(t)^T \quad \dot{y}_{ref}(t)^T \quad \ddot{y}_{ref}(t)^T \quad ... \quad y_{ref}^{[r]}(t)^T \right]$$

2/ Optimal control

Based on the previous development, the optimal control law can be derived as:

$$\dot{u}(t) = -(D_u D_{f_x}^{\mu} h(x))^{-1}(K M_{\mu+1}(t) + D_{f_x}^{\mu+1} h(x) - y_{ref}^{[\mu+1]}(t)) \quad \text{(A.7)}$$

Where $M_{\mu+1}(t) \in \Re^{\mu+1}$ is given by :

$$M_{\mu+1} = \begin{bmatrix} h(x) - y_{ref}(t) \\ D_{f_x}^{1} h(x) - \dot{y}_{ref}(t) \\ \vdots \\ D_{f_x}^{\mu} h(x) - y_{ref}^{[\mu]}(t) \end{bmatrix} \quad \text{(A.8)}$$

and $K \in \Re^{\mu+1}$ is determined by :

$$k_i = m_i T_2^{-\mu-2+i} \frac{(\mu+1)!}{(i-1)!} \quad i = 1,..., \mu+1 \quad \text{(A.9)}$$

where $[m_1, m_2,..., m_{\mu+1}]$ denote the first row of the matrix $\overline{M}_n^{-1} \overline{M}_m$ and

$$\overline{M}_m = \begin{bmatrix} \frac{1-\frac{T_1^{\mu+2}}{T_2}}{\mu+2} & \cdots & \frac{1-\frac{T_1^{2\mu+2}}{T_2}}{2\mu+2} \\ \vdots & & \vdots \\ \frac{1-\frac{T_1^{r+1}}{T_2}}{r+1} & \cdots & \frac{1-\frac{T_1^{r+\mu+1}}{T_2}}{r+\mu+1} \end{bmatrix} \quad \text{(A.10)}$$

$$\overline{M}_n = \begin{bmatrix} \frac{1-\frac{T_1^{2\mu+3}}{T_2}}{2\mu+3} & \cdots & \frac{1-\frac{T_1^{r+\mu+2}}{T_2}}{r+\mu+2} \\ \vdots & & \vdots \\ \frac{1-\frac{T_1^{r+\mu+2}}{T_2}}{r+\mu+2} & \cdots & \frac{1-\frac{T_1^{2r+1}}{T_2}}{2r+1} \end{bmatrix} \quad \text{(A.11)}$$

B Derivative of augmented predictive error

$$\xi = \int_0^T t(\tau)\, d\tau \ \ (\hat{z} - z_m) \quad \text{(B.1)}$$

If we let

$$N = \int_{T_1}^{T_2} t(\tau) d\tau = [(T_2 - T_1), \frac{T_2^2 - T_1^2}{2}, ..., \frac{T_2^{\mu+1} - T_1^{\mu+1}}{(\mu+1)!}] \quad \text{(B.2)}$$

then $\xi = N(\hat{z} - z_m)$. (B.3)

Consider the following error vector $e = \hat{z} - z_m$ (B.4)

then $\dot{e} = \dot{\hat{z}} - \dot{z}_m$ (B.5)

If we consider

$$\hat{v} = y_{ref}^{[\mu+1]} - \sum_{i=1}^{\mu} k_i (D_{f_x}^{i} h(x) - y_{ref}^{[i]}) \quad \text{(B.6)}$$

the derivative of the predictive controller can be written as $\dot{u} = \alpha(x,\theta) + \beta(x,\theta)v$ (B.7)

Where $\alpha = -(D_u D_{f_x}^{\mu} h(x))^{-1} D_{f_x}^{\mu+1} h(x)$

and $\beta = (D_u D_{f_x}^{\mu} h(x))^{-1}$

and the model (3.1) can be written as :

$$\dot{z} = A z + b v \quad \text{(B.8)}$$

Where

$$A = \begin{bmatrix} 0 & 1 & 0 & ... & & 0 \\ 0 & 0 & 1 & ... & & 0 \\ & & . & . & & \vdots \\ & & & . & . & \\ 0 & 0 & 0 & ... & . & 1 \\ 0 & 0 & 0 & ... & & 0 \end{bmatrix}; \quad b = \begin{bmatrix} 0 \\ : \\ : \\ 0 \\ 1 \end{bmatrix} \quad \text{(B.9)}$$

The vector $\hat{z}$ (3.5) can be expressed as

$$\hat{z}_1 = \hat{h}\,(x)$$

$$\hat{z}_2 = D_{f_0}\,\hat{h}\,(x) + \sum_{i=1}^{q} D_{f_i}\,\hat{h}(x)\,\hat{\theta}_i$$

$$\hat{z}_3 = D_{f_0}^2\hat{h}\,(x) + \sum_{i=1}^{q}\left(D_{f_0}D_{f_i} + D_{f_i}D_{f_0}\right)\hat{h}(x)\,\hat{\theta}_i + \sum_{i=1}^{q}\sum_{j=1}^{q} D_{f_i}D_{f_j}\,\hat{h}\,(x)\,\hat{\theta}_i\,\hat{\theta}_j \qquad \text{(B.10)}$$

$$\vdots$$

Then

$$\dot{\hat{z}}_1 = D_{f_x}\,\hat{h}\,(x) + \frac{\partial\hat{h}}{\partial\hat{\theta}}\,\dot{\hat{\theta}}$$

$$= D_{f_0}\hat{h}\,(x) + \sum_{i=1}^{q} D_{f_i}\hat{h}(x)\,\theta_i + \frac{\partial\hat{h}}{\partial\hat{\theta}}\,\dot{\hat{\theta}} \qquad \text{(B.12)}$$

$$= \hat{z}_2 + \sum_{i=1}^{q} D_{f_i}\hat{h}\left(\theta_i - \hat{\theta}_i\right) + \frac{\partial\hat{h}}{\partial\hat{\theta}}\,\dot{\hat{\theta}}$$

Similarly, we can write for $\hat{z}_i$, $i = 2,\ldots,\mu$:

$$\dot{\hat{z}}_i = \frac{dD_{f_x}^{\,i-1}\,\hat{h}}{dt} \qquad \text{(B.13)}$$

$$= D_{f_0}D_{\hat{f}_x}^{\,i-1}\hat{h} + \sum_{i=1}^{q} D_{f_i}D_{\hat{f}_x}^{\,i-1}\hat{h}\,\theta_i + \frac{\partial D_{\hat{f}_x}^{\,i-1}\hat{h}}{\partial\hat{\theta}}\,\dot{\hat{\theta}}$$

Recall that

$$\hat{z}_{i+1} = D_{\hat{f}_x}^{i}\,\hat{h} = D_{f_0}D_{\hat{f}_x}^{i-1}\,\hat{h} + \sum_{i=1}^{q} D_{f_i}D_{\hat{f}_x}^{i-1}\,\hat{h}\,\hat{\theta}_i \qquad \text{(B.14)}$$

Therefore $\dot{\hat{z}}_i$ can be written as

$$\dot{\hat{z}}_i = \hat{z}_{i+1} + \sum_{i=1}^{q} D_{f_i}\,D_{\hat{f}_x}^{\,i-1}\,\hat{h}\left(\theta_i - \hat{\theta}_i\right) + \frac{\partial D_{\hat{f}_x}^{\,i-1}\hat{h}}{\partial\hat{\theta}}\,\dot{\hat{\theta}} \qquad \text{(B.15)}$$

for $i=\mu+1$

$$\dot{\hat{z}}_{\mu+1} = \frac{d\,D_{\hat{f}_x}^{\mu}\,\hat{h}}{dt}$$

$$= D_{f_0}D_{\hat{f}_x}^{\mu}\,\hat{h}(x) + \sum_{i=1}^{q} D_{f_i}D_{\hat{f}_x}^{\mu}\,\hat{h}(x)\,\theta_i + \left[D_{u_0}D_{\hat{f}_x}^{\mu}\,\hat{h} + \sum_{i=1}^{q} D_{u_i}D_{\hat{f}_x}^{\mu}\,\hat{h}\,\theta_i\right]\dot{\hat{u}} + \frac{\partial D_{\hat{f}_x}^{\mu}\,\hat{h}}{\partial\hat{\theta}}\,\dot{\hat{\theta}} \qquad \text{(B.16)}$$

Applying the controller

$$\dot{\hat{u}} = \frac{1}{D_u D_{\hat{f}_x}^{\mu}\,\hat{h}}\left(\hat{v} - D_{\hat{f}_x}^{\mu+1}\,\hat{h}\right), \text{ then} \qquad \text{(B.17)}$$

$$\dot{\hat{z}}_{\mu+1} = \sum_{i=1}^{q} D_{\hat{f}_i}D_{\hat{f}_x}^{\mu}\hat{h}\,\left(\theta_i - \hat{\theta}_i\right) + \hat{v} + \sum_{i=1}^{q} D_{\hat{u}_i}D_{\hat{f}_x}^{\mu}\,\hat{h}\,\left(\theta_i - \hat{\theta}_i\right)\dot{\hat{u}} + \frac{\partial D_{\hat{f}_x}^{\mu}\,\hat{h}}{\partial\hat{\theta}}\,\dot{\hat{\theta}} \qquad \text{(B.18)}$$

If we regroup (B.13…B.19) and if we let $\Delta\theta = \theta - \hat{\theta}$ (B.19)

Then we can write

$$\dot{\hat{z}} = A\,\hat{z} + b\,\hat{v} + W_1\left(x,\hat{\theta},\hat{v}\right)\Delta\theta + W_2\left(x,\hat{\theta}\right)\dot{\hat{\theta}} \qquad \text{(B.20)}$$

Where

$$W_1\left(x,\hat{\theta},v\right) = \begin{pmatrix} D_{f_1}\hat{h} & \ldots & D_{f_q}\hat{h} \\ D_{f_1}D_{\hat{f}_x}\hat{h} & \ldots & D_{f_q}D_{\hat{f}_x}\hat{h} \\ \vdots & & \vdots \\ \vdots & & \vdots \\ D_{f_1}D_{\hat{f}_x}^{\mu-1}\hat{h} & \ldots & D_{f_q}D_{\hat{f}_x}^{\mu-1}\hat{h} \\ D_{f_1}D_{\hat{f}_x}^{\mu}\hat{h} + D_{u_i}D_{\hat{f}_x}^{\mu}\hat{h}\hat{u} & \ldots & D_{f_q}D_{\hat{f}_x}^{\mu}\hat{h} + D_{u_q}D_{\hat{f}_x}^{\mu}\hat{h}\hat{u} \end{pmatrix} \qquad \text{(B.21)}$$

and $W_2\left(x,\hat{\theta}\right) = \dfrac{\partial\hat{z}\left(x,\hat{\theta}\right)}{\partial\hat{\theta}}$ (B.22)

The form (B.21) without the term $W_2\,\dot{\hat{\theta}}$ is familiar in the linear adaptive control literature [10], [11]. The quantity multiplying the parameter error $\Delta\theta, W_1(x,\hat{\theta},v)$ is often referred to as the regressor. The existence of the nonlinear term $W_2\dot{\hat{\theta}}$ suggests the use of the concept of augmented error, defined in linear adaptive control [11-14].

In the design of a model reference adaptive controller for a linearised system, the reference model must be asymptotically stable and continuously derivable until order larger than or equal to $\mu+1$. We consider the following model reference,

$$\dot{z}_m = \begin{pmatrix} \dot{y}_{ref} \\ \ddot{y}_{ref} \\ \vdots \\ y_{ref}^{[\mu]} \\ y_{ref}^{[\mu+1]} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & .. & 0 \\ 0 & 0 & 1 & .. & 0 \\ \vdots & \vdots & \vdots & .. & \vdots \\ 0 & 0 & 0 & .. & 1 \\ -k'_0 & -k'_1 & -k'_2 & .. & -k'_\mu \end{pmatrix}\begin{pmatrix} y_{ref} \\ \dot{y}_{ref} \\ \vdots \\ \vdots \\ y_{ref}^{[\mu]} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} r \qquad \text{(B.23)}$$

Yet

$$\begin{cases} \dot{z}_m = A_m\,z_m + b_m\,r \\ z_m(0) = z_{m0} \end{cases} \qquad \text{(B.24)}$$

The signal $r(t)$ is a uniformly bounded input, and z_m is a prescribed reference trajectory we wish the state z to track. The spectrum of A_m lies in the negative left half plane, and the pair (A_m , b_m), without loss of generality, is assumed to be in controllable form.

The derivative of error e is then defined as :

$$\dot{e} = \dot{\hat{z}} - \dot{z}_m \qquad \text{(B.25)}$$

$$= A\hat{z} + b\hat{v} + W_1\Delta\theta + W_2\,\dot{\hat{\theta}} - A_m z_m - b_m\,r$$

$$\dot{e} = A\hat{z} + W_1\Delta\theta + W_2\,\dot{\hat{\theta}} - A\,z_m + \quad \text{(B.26)}$$

$$b\,[\,y_{ref}^{\mu+1} - \sum_{i=0}^{\mu} k_i\,(\,D_f^i h - y_{ref}^{i}\,) - r - \sum_{i=0}^{\mu}(-k'_i)\;y_{ref}^{i}]$$

Let $A_n = A + b\,K$ (B.27)

Where $K = [k_0, k_1, \ldots.., k_\mu]$ then

$$\dot{e} = A_n\,e + W_1\Delta\theta + W_2\,\dot{\hat{\theta}} + b[\,y_{ref}^{\mu+1} - r - \sum_{i=0}^{\mu}(-k'_i)\,y_{ref}^{i}] \quad \text{(B.28)}$$

By using (B.24) $y_{ref}^{[\mu+1]}$ can be written as

$$y_{ref}^{\mu+1} = \sum_{i=0}^{\mu}(-k'_i)\;y_{ref}^{i} + r \quad \text{(B.29)}$$

Then $\dot{e} = A_n\,e + W_1\Delta\theta + W_2\,\dot{\hat{\theta}}$ (B30)

Note that $\xi = N\,(\hat{z} - z_m) = N\,e$ is a scalar containing a sum of the following terms :

$$\xi = (T_2 - T_1)\,e_1 + \frac{T_2^2 - T_1^2}{2!}e_2 + \ldots\ldots \quad \text{(B.31)}$$

In the Squeal, we shall assume that if the terms $(T_2 - T_1)e_1, \frac{T_2^2 - T_1^2}{2!}e_2 \ldots..$ have minimal values the expression of ξ is minimal. Let us consider an augmented predictive error ξ_a defined by $\xi_a = M\;\;e$ (B.32)

where

$$M = \begin{pmatrix} T_2 - T_1 & 0 & . & . & 0 \\ 0 & \frac{T_2^2 - T_1^2}{2!} & . & . & . \\ . & . & \frac{T_2^3 - T_1^3}{3!} & . & . \\ . & & . & . & 0 \\ 0 & . & . & 0 & \frac{T_2^{\mu+1} - T_1^{\mu+1}}{(\mu+1)!} \end{pmatrix}$$

diagonal matrix formed with the element of matrix N.

Performing the minimisation of ξ_a implies the minimisation of ξ.

$$\begin{aligned} \dot{\xi}_a &= M\;\dot{e} = M\;A_n\,e + M\;W_1\Delta\theta + M\,W_2\,\dot{\hat{\theta}} \\ &= M\;A_n M^{-1}\,M\;e + W'_1\Delta\theta + W'_2\,\dot{\hat{\theta}} \\ &= M\;A_n M^{-1}\,\xi_a + W'_1\Delta\theta + W'_2\,\dot{\hat{\theta}} \\ &= A'_n\,\xi_a + W'_1\Delta\theta + W'_2\,\dot{\hat{\theta}} \end{aligned} \quad \text{(B.33)}$$

Where $A'_n = M\;A_n\,M^{-1}$ (B.34)

$W_1' = M\;W_1$ and $W_2' = M\,W_2$ (B.35)

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