Second Order Sliding Mode Control of Nonlinear Multivariable Systems

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***Abstract*— The paper proposes an approach to second order sliding mode control for multi-input multi-output (MIMO) nonlinear uncertain systems. With respect to standard sliding mode control, the second order sliding mode techniques for single-input single-output (SISO) systems show the same properties of robustness and precision, feature a higher order accuracy and can be exploited to eliminate the chattering effect. The extension of these results to the MIMO nonlinear systems is a challenging matter. In the present paper, the validity is extended to a quite large class of nonlinear processes affected by uncertainties of general nature; the control design is simple; the conditions of existence on the controllers are weak. The proposed procedure represents a general approach to the second order sliding mode control of MIMO systems.**

I. INTRODUCTION

In order to overcome the problem of chattering in classical sliding mode control, a new approach called “higher order sliding mode” has been recently proposed [1], [2], [3]. In first order sliding mode control the sliding variable is selected such that it has relative degree one with respect to the control. The discontinuous control signal acts on $\dot{s}$, the first time derivative of the sliding output s, to enforce a sliding motion on $s = 0$. The concept of higher order sliding modes has been developed as the generalization of first order sliding modes: the control acts on the higher derivatives of s. For example, in the second order sliding modes, the control affects $\ddot{s}$, the second derivative of the sliding variable. The higher order sliding mode control provides a natural solution to avoid the chattering effect [4], shows robustness against various kinds of uncertainties such as external disturbances and measurement errors (as the standard sliding mode control [5]) and provides a higher order precision [1]. The main results concern the second order sliding mode control [1], [3], [6], even if sliding mode strategies of order higher than two have been proposed for SISO nonlinear systems [7], [8], [9], [10].

The extension of these results to the MIMO nonlinear systems is a challenging matter. Very few results on the higher order sliding mode control for MIMO nonlinear systems have been presented, mainly due to the non-applicability of Lyapunov’s direct method. Some existing results, [11], [12], [13], apply to second order sliding mode control systems with a “low” coupling between the inputs and the sliding variables. However, this condition is quite restrictive (for example it does not apply to general Lagrangian systems). For the class of Lagrangian systems solutions are given in [14] and [11]; they require the exploitation of observers, rely on the hierarchical sliding mode concept and ensure asymptotic convergence. The solution in [12] is based on regular form, while the one in [13] is based on quite restrictive conditions. In [15] a generalization of [10] is proposed by using the restrictive “low” coupling condition.

The present paper proposes a MIMO second order sliding mode strategy, the validity of which is extended to a quite large class of nonlinear processes affected by uncertainties of general nature (for example, but not only, the Lagrangian systems). In Section II the “suboptimal” second order sliding mode algorithm for SISO control systems is briefly introduced. The considered MIMO second order control systems are described in the following section. In Section IV the proposed second order sliding mode control strategy for multivariable nonlinear uncertain systems is presented in details. The control design is simple; the conditions of existence on the controllers are weak. The proposed procedure represents a general approach to the second order sliding mode control of MIMO systems. Two examples are introduced and simulation results are shown.

II. SISO SECOND ORDER SLIDING MODE CONTROL SYSTEMS

Consider an uncertain single-input nonlinear system

$$\dot{x} = f(t, x, u), \quad t \geq 0, \tag{1}$$

where $x \in X \subseteq I\!R^n$ is the measurable state vector, $u \in U \subset I\!R$ is the bounded control input and $f(t, x, u) : [0, +\infty) \times I\!R^n \times U \rightarrow I\!R^n$ is a sufficiently smooth uncertain vector field.

Define a proper sliding manifold in the state space

$$s(t, x) = 0, \tag{2}$$

where $s(t, x) : [0, +\infty) \times X \rightarrow I\!R$ is a known, single valued function.

The control objective is that of steering to zero, possibly in a finite time, the measurable sliding output $s(t, x)$.

Assume that the sliding variable has a globally defined relative degree two [16], which implies that the second derivative of s can be expressed as

$$\ddot{s} = \varphi(t, x) + \gamma(t, x)u, \tag{3}$$

where the control gain function γ is always separated from zero. It can often be assumed that the sign of γ is known and that, under sensible conditions, the uncertainties of the system are bounded.

The effective application of Lyapunov-based or classical first order sliding mode control design methods ([17], [18]) to system (1) with (2) is possible. The main drawbacks are when the relative degree p of the measurable regulated quantity s is higher than one; the control methods generally require the knowledge of the derivatives of s up to the $(p-1)$-th order. As far as $p = 2$, the usually nonmeasurable $\dot{s}$ must be estimated by means of some observer (e.g., "high-gain" observer [17] or sliding differentiator [19]).

If the state trajectory of system (1) lies on the intersection of the two manifolds $s = 0$ and $\dot{s} = 0$ in the state space, then system (1) evolves featuring a second order sliding mode on the sliding manifold (2), [20].

A. Second Order Sliding Mode Control for SISO Systems

The sliding variable s can be considered as a suitable output of the uncertain system (1); the control aim is that of steering this output to zero in a finite time. The second order sliding mode approach allows for the finite time stabilization of both the output variable s and its time derivative $\dot{s}$ by defining a suitable discontinuous control function which can be either the actual plant control or its time derivative, depending on the system relative degree [6].

Let us set $y_1 = s$, it has been shown that, under sensible conditions, apart from a possible initialization phase, the second order sliding mode problem is equivalent to the finite time stabilization problem for the following uncertain second order system [6]

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= \varphi(t, y) + \gamma(t, y)v, \end{aligned} \tag{4}$$

where y_2 is not available, but its sign is known. The uncertain functions $\varphi(t, y)$ and $\gamma(t, y)$ are such that

$$\begin{aligned} |\varphi(t, y)| &< \Phi, \\ 0 < \Gamma_1 < \gamma(t, y) &< \Gamma_2, \end{aligned} \tag{5}$$

for all $y \in Y \subseteq I\!R^2$, such that the uncertain dynamics (4) is bounded. If system (1) has relative degree $p = 2$ with respect to $y_1 = s$, then $v = u$, while, if $p = 1$, $v = \dot{u}$.

The second order sliding mode approach [6] solves the stabilization problem for (4) by requiring the knowledge of y_1 and just the sign of y_2. Some algorithms, which propose a solution to the above control problem, have been presented in the literature [1], [3], [21], [6]. In the present paper the focus is on the so-called "suboptimal" second order sliding mode algorithm which derives from a suboptimal feedback implementation of the classical time optimal control for a double integrator [3], [21].

The class of "suboptimal" second order sliding mode algorithms can be defined by the following control law

$$v(t) = -\alpha(t) V \text{sign}\left(y_1 - \beta y_{1M_i}\right), \tag{6}$$

$$\alpha(t) = \begin{cases} 1 & \text{if} \quad (y_1 - \beta y_{1M_i})\, y_{1M_i} \geq 0, \\ \alpha^* & \text{if} \quad (y_1 - \beta y_{1M_i})\, y_{1M_i} < 0. \end{cases}$$

where $V > \max\left(\frac{2\Phi}{\Gamma_1}, \frac{2\Phi}{(1+\beta)\alpha^*\Gamma_1 + (\beta-1)\Gamma_2}\right)$ is the control magnitude parameter, $\alpha^* > \max\left(1, \frac{(1-\beta)\Gamma_2}{(1+\beta)\Gamma_1}\right)$ is the modulation parameter and $\beta \in [0, 1)$ is the anticipation parameter; y_{1M_i} is the last "singular point" of the sliding output y_1 (i.e., the value of s at the most recent time instant at which $\dot{s}$ is zero). By properly setting the controller parameters V, α^* and β, it turns out that both s and $\dot{s}$ converge to zero in finite time. Moreover some stability and transient requirements can be obtained (this possibility can be exploited in many mechanical applications in order to avoid impacts and stick-slip phenomena [22]). The discontinuous control law is given by (6), except in a possible initialization phase [3], [21]. The control problem is solved by (6) even when the bounds on the system are not constants and depend on the modulus of s [14], [22].

III. MIMO SECOND ORDER SLIDING MODE CONTROL SYSTEMS

Consider an uncertain multi-input nonlinear system

$$\dot{x} = f(t, x, u), \quad t \geq 0, \tag{7}$$

where $x \in X \subseteq I\!R^n$ is the measurable state vector, $u \in U \subset I\!R^k$ is the bounded vector of the control inputs and $f(t, x, u) : [0, +\infty) \times I\!R^n \times U \rightarrow I\!R^n$ is a sufficiently smooth uncertain vector field.

Define a proper sliding manifold in the state space

$$s(t, x) = 0, \tag{8}$$

where $s(t, x) : [0, +\infty) \times X \rightarrow I\!R^m$ is a known vector function.

The control objective is that of steering to zero, possibly in a finite time, each component of the measurable sliding vector $s(t, x)$.

In this paper we consider the nominal system (7), (8) with a special structure, as follows. Let n be even and partition the state variable as

$$x = (q', p')', \quad q \in I\!R^j, \quad j = \frac{n}{2}. \tag{9}$$

We assume that $j = k = m$ and

$$f(t, x, u) = \begin{bmatrix} p \\ A(x) \end{bmatrix} + \begin{bmatrix} 0_{m\times m} \\ B(q) \end{bmatrix} u, \tag{10}$$

$$s(t, x) = Cx, \tag{11}$$

where $A(x) \in I\!R^m$, $B(q) \in I\!R^{m\times m}$ and C is a constant $m \times 2m$ matrix. We assume, for sake of simplicity, that s and the dynamics are independent of time.

The special structure (9), (10), (11) is motivated by applications to mechanical systems for which q is the vector of the Lagrangian coordinates and $p = \dot{q}$. We emphasize that the matrix B in (10) does not depend on p and is positive definite.

In a finite time the measurable sliding output $s(x)$ must be steered to zero by means of the control vector u.

Let us choose $C = [\mathrm{I}_{m\times m}\, 0_{m\times m}]$, the second time derivative of s can be expressed as

$$\ddot{s} = A(s, \dot{s}) + B(s)u. \tag{12}$$

The following conditions on the uncertain $B(s)$ and $A(s,\dot{s})$ hold

$$0<\lambda_1 \le \lambda_{min} \le \lambda_{max} \le \lambda_2, \tag{13}$$

$$\|A(s,\dot{s})\| \le \mathcal{A}\,\|s\|\,, \tag{14}$$

where λ_{min} and λ_{max} are, respectively, the minimum and maximum eigenvalue of $B(s)$, while λ_1, λ_2, $\mathcal{A}$ are known positive constants.

IV. SECOND ORDER SLIDING MODE CONTROL FOR MIMO SYSTEMS

Let us consider the following scalar function

$$W\left(s\right)=\frac{1}{2}s's=\frac{1}{2}\left\|s\right\|^2, \tag{15}$$

by definition $W\left(s\right)$ is positive definite; its first time derivative is given by

$$\dot{W}\left(t\right)=s'\dot{s} \tag{16}$$

while the second time derivative is

$$\ddot{W}\left(t\right)=\dot{s}'\dot{s}+s'A(s,\dot{s})+s'B(s)u. \tag{17}$$

By posing $y_1=W$, we obtain

$$\begin{array}{l}\dot{y}_1=y_2,\\ \dot{y}_2=F(s,\dot{s})+G(s)u,\end{array} \tag{18}$$

where the uncertain terms are $F(s,\dot{s})=\dot{s}'\dot{s}+s'A(s,\dot{s})$ and $G(s)=s'B(s)$.

The original MIMO control problem is solved if we are able to steer to zero y_1 of system (18).

Definition 1: Consider system (18) at the time instants

$$t_{M_i}:\; y_2\left(t_{M_i}\right)=0,\; i=1,2,\ldots \tag{19}$$

and define the singular points of the variable y_1 as

$$y_{1M_i}=y_1\left(t_{M_i}\right),\; i=1,2,\ldots, \tag{20}$$

by (15) $y_{1M_i}\ge 0$, $i=1,2,\ldots$.

Definition 2: At the time instants (19), the variable y_1 assumes the values

$$y_{1M_i}=y_1\left(t_{M_i}\right)=\frac{1}{2}\left[s_1^2\left(t_{M_i}\right)+\ldots+s_m^2\left(t_{M_i}\right)\right]. \tag{21}$$

Define the extremal points of the sliding vector s as

$$\begin{array}{rcl}s_{M_i}&=&\left[s_1\left(t_{M_i}\right),\ldots,s_m\left(t_{M_i}\right)\right]'\\ &=&\left[s_{1M_i},\ldots,s_{mM_i}\right]',\; i=1,2,\ldots.\end{array} \tag{22}$$

Nothing can be said on the sign and the value assumed by the extremal points s_{lM_i}, $l=1,\ldots,m$, $i=1,2,\ldots$.

Proposition 1: Consider system (18) at $t=0$ and assume that $y_1\left(0\right)$ is not a singular point, i.e. $y_2\left(0\right)\neq 0$. The application of the control vector

$$u=-\mathcal{U}\mathrm{sign}\,\left(\dot{s}\right), \tag{23}$$

with $\mathcal{U}>\frac{\mathcal{A}\|s\|}{\lambda_1}$, guarantees that an extremal point is reached in a finite time.

Proof. See the Appendix.

Theorem 1: Consider system (18) at $t=t_{M_1}$ and let the control vector u, for $t\in[t_{M_1},+\infty)$, be designed according to the following

$$|u_l|=\rho(t)V, \quad l=1,\ldots,m, \tag{24}$$

$$\rho(t)=\left\{\begin{array}{ll}\frac{1}{\sqrt{m}} & \text{if } \left(y_1-\beta y_{1M_i}\right)y_{1M_i}\ge 0,\\ \frac{\alpha^*}{\sqrt{m}} & \text{if } \left(y_1-\beta y_{1M_i}\right)y_{1M_i}<0,\end{array}\right. \tag{25}$$

$$\mathrm{sign}\,\left(u\right)=-\mathrm{sign}\,\left(y_1-\beta y_{1M_i}\right)\mathrm{sign}\,\left(s\right). \tag{26}$$

The control parameters are chosen

$$\begin{array}{l}V>0, \quad V>\max\left(\frac{2\mathcal{A}\|s_{M_1}\|}{\lambda_1+4(\beta-1)\lambda_2},\frac{2\mathcal{A}\|s_{M_1}\|}{(1+\beta)\alpha^*\lambda_1+5(\beta-1)\lambda_2}\right),\\ \qquad \alpha^*>\max\left(1,\frac{5(1-\beta)\lambda_2}{(1+\beta)\lambda_1}\right), \quad \beta\in[0,1)\,.\end{array} \tag{27}$$

In a finite time, both y_1 and y_2 are steered to zero; it turns out that both s and $\dot{s}$ converge to zero in finite time.

Proof. See the Appendix.

Remark 1: It can be noticed that, when $\|s\|=0$, the control matrix $G\left(s\right)$ of system (18) is zero, nevertheless the controllability of the system is not lost since the control matrix of (12) is $B\left(s\right)>0$.

Remark 2: The MIMO second order sliding mode control strategy expressed by (24)–(26) is robust with respect to the uncertainties, the norm of which can be upper-bounded. Even in the MIMO case, the knowledge of the bounds (13) and (14) on the uncertain terms results sufficient to compute the control parameters according to (27).

V. EXAMPLES

This section presents two examples in which the previously introduced MIMO second order sliding mode procedure is applied. The control strategy generalizes the concept of MIMO first order sliding mode to the second order. The designed control vectors are discontinuous and act on the second time derivative of the sliding outputs, which are steered to zero in a finite time.

Example 1: Let us apply the proposed MIMO second order sliding mode procedure to the control system

$$\dot{x}=\left[\begin{array}{c}\dot{q}\\ \dot{p}\end{array}\right]=\left[\begin{array}{c}p\\ A\left(x\right)\end{array}\right]+\left[\begin{array}{c}0_{2\times 2}\\ B\left(q\right)\end{array}\right]u, \tag{28}$$

$$s(x)=\left(x_1\; x_2\right)', \tag{29}$$

where $x\in I\!R^4$, $q\in I\!R^2$, $p\in I\!R^2$, $u\in I\!R^2$, $A\left(x\right)=\left(x_1\; 0\right)'$ and $B\left(q\right)=[7\;3;\;3\;5]$.

The sliding vector s is steered to zero in a finite time (Figure 1); the application of a discontinuous control vector u designed according to the proposed MIMO second order sliding mode control procedure (Figure 2) generates a strictly contractive sequence $\{y_{1M_i}\}$ (Figure 3).

Example 2: The previously introduced MIMO second order sliding mode control procedure is applied to the system

$$\dot{x}=\left[\begin{array}{c}\dot{q}\\ \dot{p}\end{array}\right]=\left[\begin{array}{c}p\\ A\left(x\right)\end{array}\right]+\left[\begin{array}{c}0_{2\times 2}\\ B\left(q\right)\end{array}\right]u, \tag{30}$$

$$s(x)=\left(x_1\; x_2\right)', \tag{31}$$

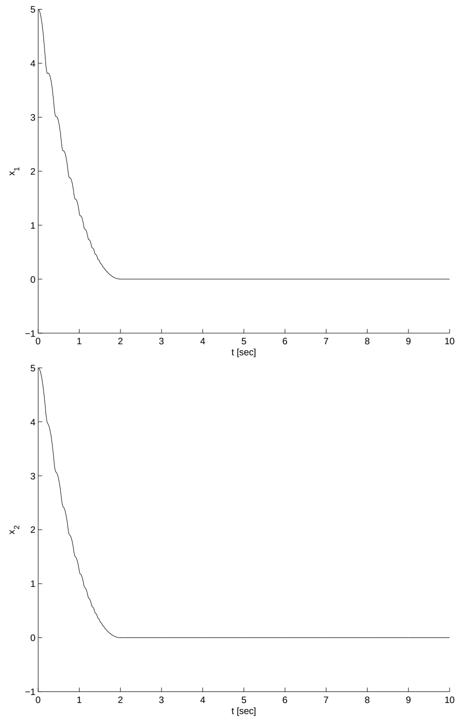

Fig. 1. In a finite time, x_1 and x_2 converge to 0.

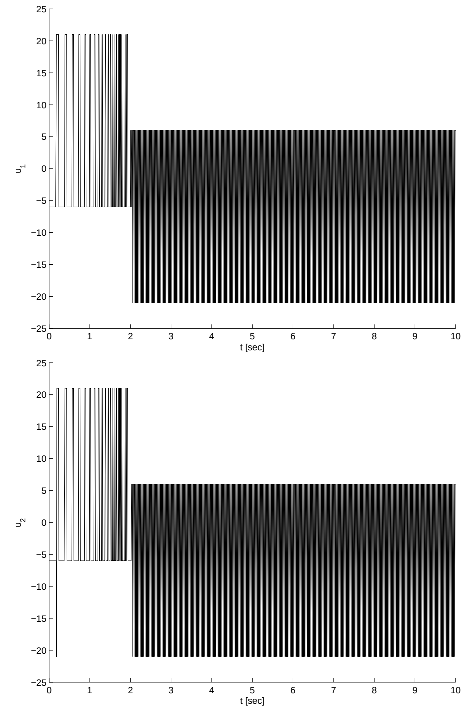

Fig. 2. The two components u_1 and u_2 of the control vector u.

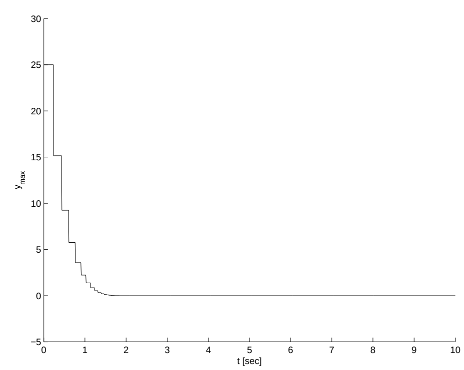

Fig. 3. In a finite time, the sequence y_{1M_i} converges to 0.

where $x \in I\!R^4$, $q \in I\!R^2$, $p \in I\!R^2$, $u \in I\!R^2$, $A(x) = (x_1 \; 0)'$ and $B(q) = [2\;3;\;3\;5]$. The matrix B in (30) is not dominant diagonal.

The sliding vector s is steered to zero in a finite time (Figure 4); the application of a discontinuous control vector u designed according to the proposed MIMO second order sliding mode control procedure (Figure 5) generates a strictly contractive sequence $\{y_{1M_i}\}$ (Figure 6).

Particularly, in this second example, the control system does not satisfy the "low" coupling condition between inputs and the sliding variables. The control problem is solved by the proposed strategy directly, without introducing any auxiliary system and without relying on a hierarchy in the reaching of the sliding manifolds [11].

VI. CONCLUSIONS

The paper proposes an approach to second order sliding mode control for MIMO nonlinear uncertain systems. The extension of the SISO second order sliding mode techniques to the MIMO nonlinear systems is a challenging matter. In this paper, the presented method applies to a quite large class of nonlinear processes affected by uncertainties of general nature, the control design is simple and the conditions of existence on the controllers are weak. The proposed procedure represents a general approach to the second order sliding mode control of MIMO systems.

APPENDIX

Proof of Proposition 1.

The control vector (23) applied to system (12), guarantees that, after a finite time and for any initial condition $\dot{s}(0)$, the inequality $\dot{s}'\ddot{s} < -h^2 \|\dot{s}\|$ is satisfied. This fact implies that $\dot{s}$ is zero in a finite time. There exists $t_0 < +\infty$ such that $\dot{s}(t_0) = 0$ and, therefore, $y_2(t_0) = 0$. □

Proof of Theorem 1.

Consider system (18) at $t = t_{M_1}$ and apply the control vector u designed according to (24)–(26). The following facts hold about the scalar term $G(s)\,u$ which acts on the system

$$\lambda_1 \|s\| V \le |G(s)\,u| \le \lambda_2 \|s\| \alpha^* V, \tag{32}$$

$$\text{sign}\,[G(s)\,u] = -\text{sign}\,(y_1 - \beta y_{1M_i}). \tag{33}$$

System (18) is controlled by means of the vector u, which is designed such that the scalar control $G(s)\,u$ results in a second order law (6).

If it is possible to set the control parameters in order to satisfy the contractive conditions, the second order control law guarantees that the two sequences $\{y_{1M_i}\}$ and $\left\{\Delta t_{M_i} = t_{M_i} - t_{M_{i-1}}\right\}$ are strictly contractive, that is

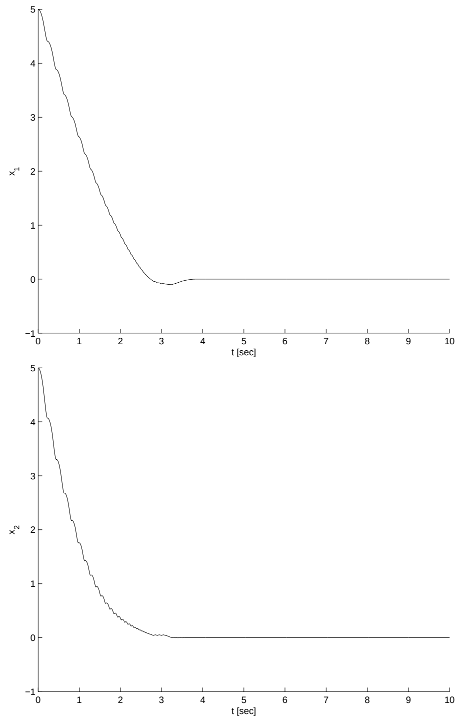

Fig. 4. In a finite time, x_1 and x_2 converge to 0.

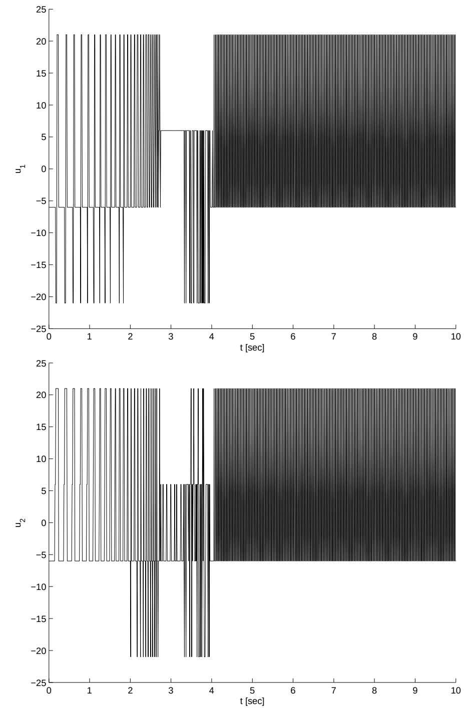

Fig. 5. The two components u_1 and u_2 of the control vector u.

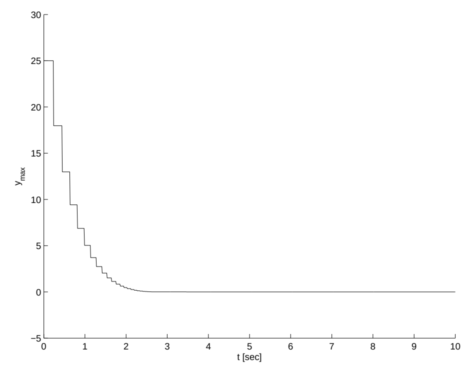

Fig. 6. In a finite time, the sequence y_{1M_i} converges to 0.

$\dfrac{y_{1M_i}}{y_{1M_{i-1}}} \leq r < 1$ and $\dfrac{\Delta t_{M_i}}{\Delta t_{M_{i-1}}} \leq q < 1$. This implies $\lim\limits_{i\to+\infty} y_{1M_i} = 0$ and $\sum_{i=1}^{+\infty} \Delta t_{M_i} = T < 1$; at $t = T$ both y_1 and y_2 are steered to zero.

In order to set the conditions on the control parameters for this MIMO case, we need to find a bound on the term $\|\dot{s}\|^2$ in $F(s, \dot{s})$.

Consider (12) at $t = t_{M_1}$ and the control vector designed by (24)–(26), it can be easily verified that

$$|F(s, \dot{s})| \leq \mathcal{D}\,\|s\| + \mathcal{A}\,\|s\|^2\,, \tag{34}$$

where the positive constant $\mathcal{D} = 2\lambda_2 V\,(1-\beta)$ depends on the choice of the control parameters and on the bounds on the uncertainties of the system; condition (34) holds for all $x \in X$, $u \in U$.

By substituting (34) in the conditions on the control parameters of (6) and further computations, the conditions (27) are found for the control parameters in the MIMO case. □

REFERENCES

[1] A. Levant, "Sliding order and sliding accuracy in sliding mode control," *Int. J. Control*, vol. 58, pp. 1247–1263, 1993.

[2] S. Emelyanov, S. Korovin, and A. Levant, "High-order sliding modes in control systems," *Computational mathematics and modelling*, vol. 7, no. 3, pp. 294–318, 1996.

[3] G. Bartolini, A. Ferrara, and E. Usai, "Output tracking control of uncertain nonlinear second-order systems," *Automatica*, vol. 33, no. 12, pp. 2203–2212, 1997.

[4] ——, "Chattering avoidance by second order sliding modes control," *IEEE Trans. Automat. Control*, vol. 43, no. 2, pp. 241–247, 1998.

[5] V. Utkin, *Sliding Modes in Control and Optimization*. Springer Verlag, 1992.

[6] G. Bartolini, A. Ferrara, A. Levant, and E. Usai, "On second order sliding mode controllers," in *Variable Structure Systems, Sliding Mode and Nonlinear Control*, ser. Lecture Notes in Control and Information Sciences, K. Young and U. Ozguner, Eds. Springer Verlag, 1999.

[7] A. Levant, "Controlling output variables via higher order sliding modes," in *Proc. of the European Control Conference, ECC'99*, Karlsruhe, Germany, 1999.

[8] ——, "Universal siso sliding-mode controllers with finite-time convergence," *IEEE Trans. Automat. Control*, vol. 49, no. 9, pp. 1447–1451, 2001.

[9] ——, "Universal output-feedback siso controller," in *Proc. of the 15th IFAC World Congress*, Barcelona, Spain, 2002.

[10] S. Laghrouche, F. Plestan, and A. Glumineau, "Higher order sliding mode control based on optimal linear quadratic control," in *Proc. of the European Control Conference, ECC'03*, Cambridge, England, 2003.

[11] G. Bartolini, A. Ferrara, E. Usai, and V. Utkin, "On multi-input chattering-free second-order sliding mode control," *IEEE Trans. Automat. Control*, vol. 45, no. 9, pp. 1711–1717, 2000.

[12] T. Floquet, J. Barbot, and W. Perruquetti, "One-chained form and sliding mode stabilization for a nonholonomic perturbed system," in *Proc. of the American Control Conference, ACC00*, Chicago, USA, 2000.

[13] A. Levant, “Mimo 2-sliding control design,” in *Proc. of the European Control Conference, ECC’03*, Cambridge, England, 2003.
[14] G. Bartolini, A. Ferrara, and E. Punta, “Multi-input second-order sliding-mode hybrid control of constrained manipulators,” *Dynamics and Control*, vol. 10, no. 3, pp. 277–296, 2000.
[15] F. Plestan, S. Laghrouche, and A. Glumineau, “Higher order sliding mode control for mimo nonlinear system,” in *Proc. of the International Workshop on Variable Structure Systems, VSS04*, Vilanova i la Geltrú, Spain, 2004.
[16] A. Isidori, *Nonlinear Control Systems*. Springer Verlag, 1995.
[17] H. Khalil, *Nonlinear Systems*. NY: Prentice Hall, 1996.
[18] S. Oh and H. Khalil, “Nonlinear output feedback tracking using high-gain observers and variable structure control,” *Automatica*, vol. 33, pp. 1845–1856, 1997.
[19] A. Levant, “Robust exact differentiation via sliding mode techniques,” *Automatica*, vol. 34, pp. 379–384, 1998.
[20] L. Fridman and A. Levant, “Higher order sliding modes as a natural phenomenon in control theory,” in *Robust Control Via Variable Structure and Lyapunov Techniques*, ser. Lecture notes in control and information sciences, F. Garofalo and L. Glielmo, Eds. London: Springer Verlag, 1996, vol. 217, pp. 107–133.
[21] G. Bartolini, A. Ferrara, and E. Usai, “Applications of a sub optimal discontinuous control algorithm for uncertain second order systems,” *Int. J. of Robust and Nonlinear Control*, vol. 7, pp. 199–320, 1997.
[22] G. Bartolini, A. Pisano, E. Punta, and E. Usai, “A survey of applications of second-order sliding mode control to mechanical systems,” *Int. J. Control*, vol. 76, pp. 875–892, 2003.